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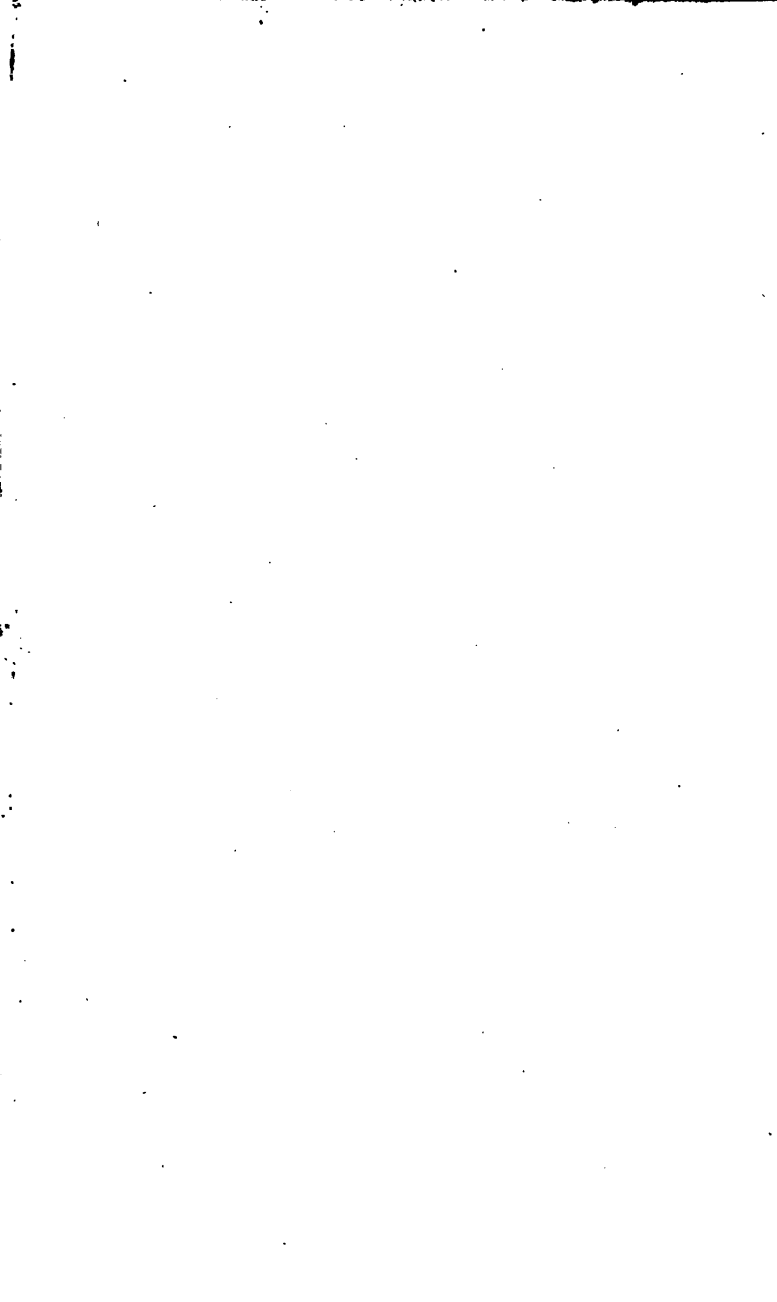
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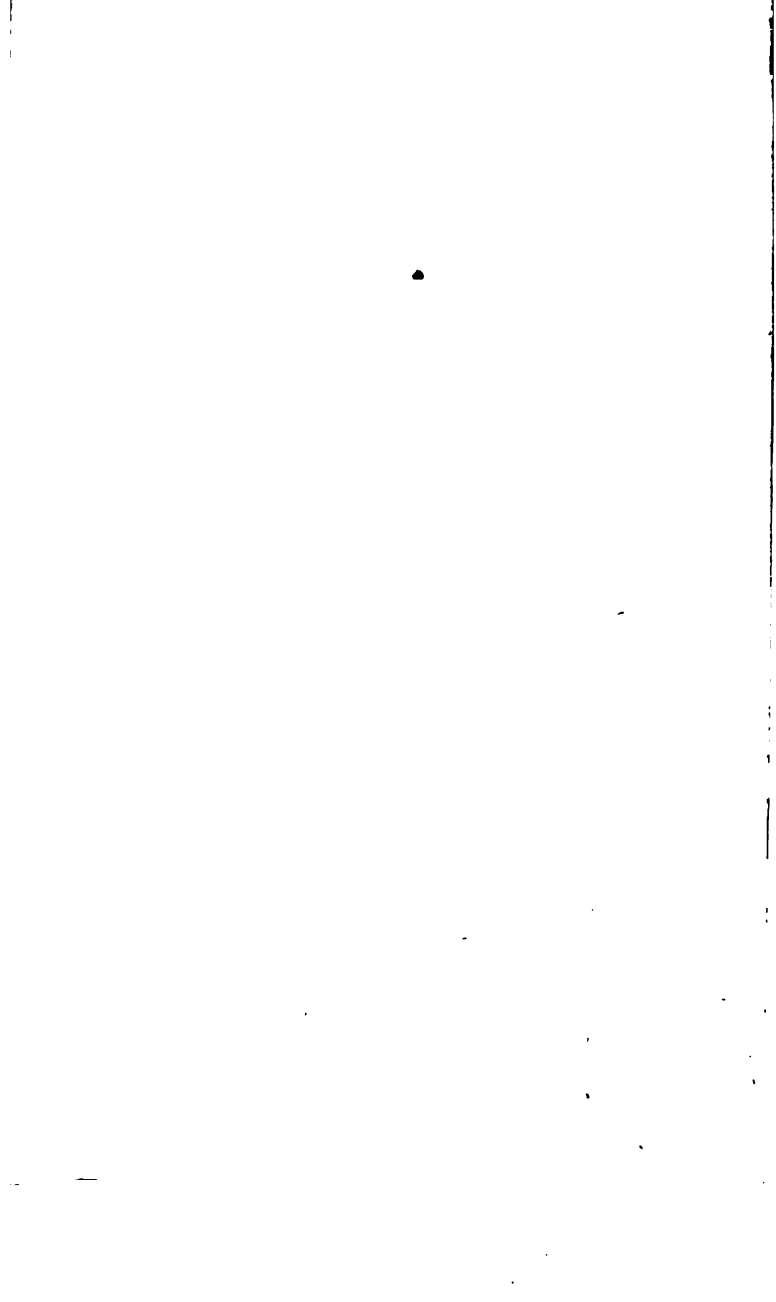
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43. 209.





PRACTICAL ARITHMETIC.



THE
PRACTICAL ARITHMETIC,
WITH
NOTES AND DEMONSTRATIONS
TO THE
PRINCIPAL RULES,
IN WHICH THE
PRINCIPLES AND PRACTICE OF THAT SCIENCE
ARE RENDERED FAMILIAR,
AND PRACTICALLY ADAPTED TO THE USE OF
PRIVATE AND PUBLIC SCHOOLS.

BY
JOHN DARBY,
TRAFALGAR STREET ACADEMY, LEEDS,
AUTHOR OF THE "STUDENT'S ALGEBRA," AND "A KEY TO THE ALGEBRAICAL
PROBLEMS OF THE REV. DR. BLAND, LATE FELLOW AND TUTOR
OF ST. JOHN'S COLLEGE, CAMBRIDGE.

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TO THE
REV. JOSEPH HOLMES D.D.
LATE FELLOW AND TUTOR OF QUEENS' COLLEGE, CAMBRIDGE,
NOW HEAD MASTER OF THE LEEDS FREE GRAMMAR SCHOOL,
IN ADMIRATION OF
HIS UNDOUBTED ATTAINMENTS IN SCIENCE AND LITERATURE,
HIS HIGH MORAL BEARING,
AND GENERAL COURTEOUS DEMEANOUR,
THIS TREATISE ON ARITHMETIC
IS,
WITH PROFOUND RESPECT,
INSCRIBED,
BY HIS MOST OBLIGED AND MOST OBEDIENT SERVANT,

JOHN DARBY.

Trafalgar Street Academy,
Leeds, January 2nd, 1842.



P R E F A C E.

THE following pages are principally deductions from the Author's manuscripts, which he has, from time to time, composed, for the use of his own Pupils; and which he now offers to the Public, under a conviction, from long experience, that the different Rules and Examples are practically adapted to the present improved state of the science.

The chapter on Mental Calculations, by which the value of any number of articles from 1 to 10,000, as well as the interest of money, may be found with as little trouble, and in most cases less, than by the use of a Ready Reckoner, merits a careful perusal.

That much neglected rule, "Profit and Loss," is reduced to actual practice. It is divided into four Cases, with practical examples to each. The questions under this head are, for the most part, taken from commercial transactions. The Table at page 155, contains an easy and concise method for finding the selling price of any commodity, in order to gain or lose a given rate per cent.

Questions similar to 9, 10, 11, and 12, in Multiplication of Vulgar Fractions, page 106, are frequently given in books on arithmetic. They may have their use; but are here only introduced to show the fallacy of their nature,

As to question 9, to multiply 2s. 6d. by 3s. 6d., correctly speaking, this is to multiply 2s. 6d. by $3\frac{1}{2}$, and not 3s. 6d. In question 10, the 17s. 6d. is multiplied by $\frac{2}{3}$, and not 13s. 4d.

Question 11, the only intelligible meaning, as De Morgan justly observes, is, if a stock of money is to be increased at the rate of £99 19s. 11 $\frac{3}{4}$ d. for every £. in it; how much would that be, when the stock itself is £99 19s. 11 $\frac{3}{4}$ d.? Similar observations will hold good for Question 12, as also Question 14 in Division. Were we to incorporate the idea of Duodecimals, we should readily conceive, that to multiply shillings by shillings, the product would be square shillings; as also, to multiply cwt. by cwt., the product would be square cwt., terms with which we are totally unacquainted.

In the Cube Root, a rule is given for finding nearly one-half of the root by Simple Division; and this portion, comprising the latter part of the figures in the root, will, in long calculations, be found particularly advantageous.

The rule for finding the number of gallons that a pump raises per stroke, will be found of use to the plumber, &c.

The work is concluded with an Appendix, containing some additional questions and illustrations on the principles of Arithmetic; and Demonstrations to the principal Rules which will form an agreeable exercise to the more advanced student.

Leeds, January 2nd, 1843.

EXPLANATION OF THE CHARACTERS

USED IN THIS WORK,

WITH WHICH THE PUPIL CANNOT BE TOO EARLY ACQUAINTED.

$+$ (*Plus*) The sign of addition signifies that the numbers between which it stands are to be added together; thus $4 + 8$, denotes that 8 is to be added to the 4.

$-$ (*Minus.*) The sign of subtraction; thus, $8 - 4$, denotes that 4 is to be subtracted from the 8.

$\times$ (*Multiple.*) The sign of multiplication; thus, 3×4 denotes that 3 is to be multiplied by the 4.

$\div$ (*Division.*) The sign of division; when placed between two quantities, signifies that the former is to be divided by the latter; thus, $8 \div 4$ denotes that the 8 is to be divided by 4.

$=$ (*Equals.*) The sign of equality generally placed between two equal quantities; thus $5 + 2 = 7$, *i. e.* 5 added to 2 is equal to 7.

$:$ $:$ $:$ (*Proportion.*) When four quantities are connected by these signs, it is called a proportion; thus, $2 : 4 :: 6 : 12$ denotes an equality of ratio, and is read, as 2 is to 4, so is 6 to 12.

$() \{ \}$ (*Brackets.*) They are employed to show that all the quantities within them are to be treated as though forming but one quantity.

$\therefore$ This sign stands for *therefore*.

$\because$ This sign is used for *because* or *since*.

$\sqrt{\quad}$ or $(\)^{\frac{1}{2}}$ denotes that the square root of the quantity which it includes is to be extracted, as $\sqrt{25}$ or $(25)^{\frac{1}{2}} = 5$.

$\sqrt[3]{\quad}$ or $(\)^{\frac{1}{3}}$ The cube root.

$\sqrt[4]{\quad}$ or $(\)^{\frac{1}{4}}$ The 4th root.

$\sqrt[n]{\quad}$ or $(\)^{\frac{1}{n}}$ The nth root, and so on.

$(\)^2$ Denotes that the quantity inclosed is to be squared, as $[25]^2$ or $(25)^2$ is equal to $25 \times 25 = 625$.

$(\)^3$ Denotes that the quantity enclosed is to be cubed.

$(\)^n$ Denotes the nth power, and so on.

ERRATA.

Line 4, page 7, for *as the*, read *as if the*.

Page 23, in the price of gold, for $10\frac{1}{4}$ d. read $10\frac{1}{2}$ d.

Quest. 10, page 45. Ans. 4 yrs. 9m. 22 days, 3 hrs.

—— 10, page 94, Ans. $\frac{49}{80}$, $\frac{39}{80}$, and $\frac{19}{80}$.

Page 119, in the note, for *add infinitum*, read *ad-infinitum*.

Quest. 16, page 121, for *decimal of a £.* read *decimal of £5.*

Quest. 11, page 180, for 07 in the answer, read 67.

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ARITHMETIC.

ARITHMETIC is the art of computing by numbers. Its leading rules are Addition, Subtraction, Multiplication, and Division.

NOTATION AND NUMERATION.

NOTATION is the method of expressing numbers.

NUMERATION is the art of reading or writing any proposed number.

The characters used in this science are called digits or figures, as follows :—

1	is called or denotes	One.
2	is	Two.
3	is	Three.
4	is	Four.
5	is	Five.
6	is	Six.
7	is	Seven.
8	is	Eight.
9	is	Nine.
0	is Nothing when taken alone, it is sometimes called Zero.	

It appears that each figure, when taken separately, expresses only so many units or ones ; but by composition any number may be expressed, as may be seen from the following table.

	Hundreds of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	2	1	9	8	7	6	5	4	3	2	1	
	2	1	9	8	0	6	5	0	3	2	0	
		2	9	0	7	6	0	4	3	0	1	
			8	7	4	5	6	7	8	3		
				1	2	3	4	5	6	7		
					3	4	5	6	7	8		
						6	4	3	2	1		
							4	3	2	1		
								5	6	7		
									2	1		
										1		

Here we may observe, that in the first line the 1 stands in the units place, and denotes one.

In the 2nd line, the 2 stands in the tenth's place, and denotes two tens or twenty; the 1 stands in the units' place, and denotes one; consequently, the 2nd line denotes twenty-one.

In the 3rd line, the 5 stands in the hundredth's place, and denotes five hundred; the 6 stands in the tenth's place, and denotes six tens or sixty; the 7 stands in the unit's place, and denotes seven; consequently, the whole line makes five hundred and sixty-seven.

In the 4th line from the bottom, the 4 stands in the thousandth's place, and denotes four thousand; consequently, the the whole line denotes four thousand three hundred and twenty-one.

In the 5th line, the 6 stands in the tens of thousandth's place, and denotes sixty thousand; therefore the whole line denotes sixty-four thousand, three hundred and twenty-one.

In the 6th line, the 3 stands in the hundreds of thousandth's place, and denotes three hundred thousand; consequently, the whole line denotes three hundred and forty-five thousand, six hundred and seventy-eight.

The top line denotes three hundred and twenty-one thousand, nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one.

Units	1
Tens	2 1
Hundreds	3 2 1
Thousands	4 3 2 1
Tens of thousands	5 4 3 2 1
Hundreds of thousands	6 5 4 3 2 1
Millions	7 6 5 4 3 2 1
Tens of millions	8 7 6 5 4 3 2 1
Hundreds of millions	9 8 7 6 5 4 3 2 1
Thousands of millions	1 9 8 7 6 5 4 3 2 1
Tens of thousands of millions	2 1 9 8 7 6 5 4 3 2 1
Hundreds of thousands of millions	3 2 1 9 8 7 6 5 4 3 2 1
Billions, (which are millions of millions)	4 3 2 1 9 8 7 6 5 4 3 2 1
Tens of billions	5 4 3 2 1 9 8 7 6 5 4 3 2 1
Hundreds of billions	6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Thousands of billions	7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Tens of thousands of billions	8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Hundreds of thousands of billions	9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Trillions	1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Tens of trillions	2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Hundreds of trillions	3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Thousands of trillions	4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Tens of thousands of trillions	5 4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Hundreds of thousands of trillions	6 5 4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Quadrillions	7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Tens of Quadrillions	8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1
Hundreds of Quadrillions	9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1 9 8 7 6 5 4 3 2 1

EXAMPLES FOR PRACTICE.

Write down in figures the following numbers :—

1. Forty-seven.
2. Four hundred and thirty-seven.
3. Fifty-six thousand two hundred and four.
4. One thousand eight hundred and forty-two.
5. Five millions, one hundred and five thousand and twenty one.
6. Two hundred and fifty-four thousand, three hundred and twenty-one millions, five hundred and one thousand, four hundred and twenty-one.

7. Two hundred and seventy-one billions, three hundred and twenty-one thousand five hundred and six millions, nine hundred and seven thousand, and twenty-five.

8. Twenty-five trillions, seventy-four billions, three hundred and fifty-one thousand two hundred and sixty-four millions, four thousand and twenty-one.

Write in words the following numbers :—

1. 321 and 1842
2. 55621
3. 706324
4. 8072346
5. 980760320
6. 123987654321
7. 123456789123456789

The Romans and other ancient nations expressed their notation or numbers, by the seven following capital letters of the alphabet :—I. V. X. L. C. D. and M.

It is still much used in the Scriptures, the Book of Common Prayer, Acts of Parliament, &c.

I.	denotes	1	XIII. or XIV.	14
V.		5	XV.	15
X.		10	XVI.	16
L.		50	XVII.	17
C.		100	XVIII.	18
D.		500	XIX.	19
M.		1000	XX.	20
II.		2	XL.	40
III.		3	LX.	60
IIII. or IV.		4	XC.	90
VI.		6	CC.	200
VII.		7	CCC.	300
VIII.		8	CD.	400
IX.		9	DC.	600
XI.		11	MM.	2000
XII.		12	MMC.	2100
XIII.		13	MDCCCXLII.	1842

SIMPLE ADDITION.

ADDITION is the art of collecting two or more numbers or quantities into one sum.

RULE.—Place the numbers to be added under each other ;

observing that the units must be placed under the units, tens under the tens, hundreds under the hundreds, and so on.

Add the figures in the column of units into one sum, and place the unit's figure of this sum under the column of units; carry the remaining figure or figures, if any, to the next column.

Add the figures in this second column, together with the number carried, into one sum as before; place the unit's figure of this sum under the second column, and carry the tens or remaining figures to the next column, and so on, as in the following example:—

Add 94567, 896, 42784, 698, 27, and 1842 into one sum.

$$\begin{array}{r}
 94567 \\
 896 \\
 42784 \\
 698 \\
 27 \\
 1842 \\
 \hline
 140814
 \end{array}$$

Here, the units 2, 7, 8, 4, 6, and 7 are placed under each other, and their sum is 34.

Now the unit's figure of this sum is 4, which must be placed under the first column, and the 3 carried or added to the next column.

Find the sum of the next column, together with the 3 which was carried; this sum is 41.

Place down the 1 under the second column, and carry the 4 to the sum of the next or third column, and so on.

The sum of the fifth or last column, together with the 1 carried to it, is 14; and as this finishes the addition, the 14 must be placed down.

The above sum or example may be decomposed as follows:

$$\begin{array}{rcl}
 \text{In the 1st row, the 9 stands for } & 90000 & \\
 \text{_____}, \text{ the 4 } & 4000 & \\
 \text{_____}, \text{ the 5 } & 500 & \\
 \text{_____}, \text{ the 6 } & 60 & \\
 \text{_____}, \text{ the 7 } & 7 & \\
 \text{In the 2nd row, the 8 } & 800 & \\
 \text{_____}, \text{ the 9 } & 90 & \\
 \text{_____}, \text{ the 6 } & 6 & \\
 \text{A } & 3 &
 \end{array}$$

In the 3rd row, the 4	stands for	40000
_____ , the 2	_____	2000
_____ , the 7	_____	700
_____ , the 8	_____	80
_____ , the 4	_____	4
In the 5th row, the 2	_____	20
_____ , the 7	_____	7
In the 6th row, the 1	_____	1000
_____ , the 8	_____	800
_____ , the 4	_____	40
_____ , the 2	_____	2

EXAMPLES FOR PRACTICE.

(1.) 246984	(2.) 367894	(3.) 1234567
267821	715968	3456789
543246	723456	2212345
456754	276544	6787655
678325	672483	6543212
321675	027517	3456788
876294	676789	9786543
123706	323212	1123456
367821	567828	6765234
<u>632178</u>	<u>432172</u>	<u>2234765</u>

NOTE.—The teacher can add as many examples as may be required to make his pupils perfect in this rule.

4. What is the sum of the following numbers:—13456, 78234, 56782, 43211, 23456, and 76544?

5. A person bought 4 bales of cotton goods; the 1st contained 325 yards, the 2nd 1200 yards, the 3rd 365 yards, and the 4th 326 yards: how many yards had he in the whole?

Ans. 2216 yards.

NOTE.—The following method of proof, though not infallible, may be given: Add the figures in each row together, beginning with the top row, rejecting the nines contained in the sum, and set all the excesses at the end under each other; find the excess of nines in this line, and also the excess of nines in the sum total; then if these excesses be the same, the work is said to be right.

43464	...	5	} excess of nines.
56926	...	4	
34848	...	0	
12345	...	6	
<u>146983</u>		<u>4</u>	Proof.

The excess of nines in the sum total would be the same if the figures

were reversed, as 389641, or if two or more figures were transposed, or if the value of one figure be too great, and that of another as much too little, or if a 9 be set down instead of a 0, or the contrary ; the excess of nines in these cases will evidently be the same as the work was right ; consequently it cannot be called a proof.

SIMPLE SUBTRACTION.

SUBTRACTION is finding the difference between any two given numbers.

RULE.—1. Place the less number under the greater, so that the units may stand under the units, tens under the tens, and so on.

2. Begin at the right or unit's number, and take each figure in the lower line, when it can be done, from that above it, and set down the remainder.

3. But if any of the figures to be subtracted be greater than the corresponding figure in the top line, add ten to the upper figure, and then take the lower figure from the sum ; observing to carry one to the next figure in the lower line, for the ten you borrowed or added, and proceed as before, till the whole be finished.

Proof.---Add the remainder to the less number, and if the sum be equal to the greater number, the work is right.

EXAMPLE UNDER THE 1ST AND 2ND PARTS.

From 873462
Take 712341

Remainder or Difference 161121

Proof 873462

EXAMPLE UNDER THE 3RD PART OF THE RULE.

From 423416
Take 281342

Rem. or Dif. 142074

Proof. 423416

Here, 2 from 6 and 4 remains ; 4 from 1 we cannot, add 10 to 1, and the sum will be 11, from which take the 4, and 7 will remain ; carry or add 1 to the 3, which will make 4,

then 4 from 4 and 0 remains ; 1 from 3 and 2 remains ; 8 from 2 we cannot, add 10 to the 2, and the sum will be 12, from which subtract 8 and 4 will remain ; carry or add one to the 2, which will make 3, then 3 from 4 and 1 remains.

EXAMPLES FOR PRACTICE.

(1.) From 3678946	(2) From 8765432
Take 1246724	Take 2331214
_____	_____
_____	_____

(3.) From 8467854	(4.) From 8761924
Take 3941916	Take 3963142
_____	_____
_____	_____

NOTE.—Any number of questions of this kind may be added by the teacher.

5. What is the difference between 7854 and 9876 ?

6. What is the sum and difference of 45678 and 79628 ?

Ans. { 125306 *sum.*
33950 *dif.*

7. A was born in the year 1794 and B in 1817 : how many years does A's age exceed that of B ? *Ans.* 23 years,

8. A was born in 1792 and died in 1841 : what was his age when he died ? *Ans.* 49 years.

9. Sir Isaac Newton was born in the year 1642 and died in 1727 : how old was he at the time of his decease ?

Ans. 85 years.

10. Homer was born 2749 years before the present year of our Lord 1842 : how many years was that before the birth of Christ ? *Ans.* 907 years.

11. Gunpowder was invented in 1344, and the Reformation commenced in 1517 : how many years were there between, and how long is it since each event, this being 1842 ?

Ans. 173 years between, and 498 and 325 since.

12. Printing* was introduced into England by Wm. Caxton in the year of our Lord 1474 ; Charles I., King of England, was beheaded January 30th, 1649 ; and Sunday schools first established in 1784 : how long is it since each event to 1842 ?

Ans. 368 years since printing was introduced.

193 ——— since Charles was martyred.

58 ——— since Sunday schools begun.

* The art of printing was invented by Laurentius of Haerlem, in the year 1430.

13. What number added to 1340961 will make two and a half millions? *Ans.* 1159039.

14. What number added to thirteen millions, thirteen hundred thousand, thirteen hundred, and thirteen will make fourteen and a half millions? *Ans.* 198687.

SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION is a short method of finding the sum of any given number, taken a certain number of times over; thus if we wish to find the sum of 5 times 365,—

$$\begin{array}{r}
 365 \\
 \underline{5} \\
 1825
 \end{array}
 \quad \text{or by} \quad
 \begin{array}{r}
 365 \\
 365 \\
 365 \\
 365 \\
 365 \\
 \hline
 1825
 \end{array}$$

Addition.

It is commonly called finding the product of two numbers; one number being called the multiplicand and the other the multiplier.

In the above example, 365 is called the multiplicand, and 5 the multiplier, and 1825 the product.

Here we may observe, that the number 365 is taken as many times as there are units in the multiplier 5. These numbers are sometimes called factors, because they are to constitute a factum or product.

THE MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The method of saying this table is as follows : twice 1 are 2, twice 2 are 4, twice 3 are 6, &c. ; three times 1 are 3, three times 2 are 6, three times 3 are 9, &c.

CASE I.—*To find the product of two numbers.*

RULE.—1. Place the multiplier under the multiplicand, so that the units may stand under the units, tens under tens, &c.

2. Begin with the unit's figure in the multiplier, and multiply each figure in the multiplicand, taking the unit's figure therein first ; putting down the whole of such product, if less than ten, under the figure multiplied by.

3. If any of the products be 10, or more, set down the unit's figure of such product, and add 1 for each 10, or carry the tens to the product of the next figure, and so on till all the figures in the multiplicand be multiplied ; observing to set down the whole product of the last or left-hand figure, as follows :—

Multiply 41962 by 4.

$$\begin{array}{r} 41962 \\ 4 \\ \hline 167848 \end{array}$$

Here $2 \times 4 = 8$, and as this 8 is less than 10, place it down ; $6 \times 4 = 24$, place the unit's figure 4 down, and carry the two to the next product, which is $9 \times 4 = 36$, and the 2 being added to 36, the sum will be 38 ; put the unit's 8 down, and carry the 3 to the next product ; $1 \times 4 = 4$, and the 3 being added to this product 4, the sum will be 7 ; now as this is less than 10, put it down ; and, lastly, $4 \times 4 = 16$, this 4 being the last figure in the multiplicand, the whole product 16 must be put down.

In the above operation—

The 2 $\times$ 4	=	8
The 6 being in the ten's place, stands for 60, then 60×4	=	240
The 9 being in the hundred's place, stands for 900, then 900×4	=	3600
The 1 being in the thousand's place, stands for 1000, then 1000×4	=	4000
The 4 being in the ten thousand's place, stands for 40000, then 40000×4	=	160000

Product 167848

Or thus :

$$\begin{array}{r}
 41962 \\
 \times 4 \\
 \hline
 8 \\
 240 \\
 3600 \\
 4000 \\
 160000 \\
 \hline
 \end{array}
 =
 \begin{array}{r}
 2 \times 4 \\
 60 \times 4 \\
 900 \times 4 \\
 1000 \times 4 \\
 40000 \times 4
 \end{array}$$

Product 167848

4. When the multiplier contains two or more figures, proceed as above with each figure in the multiplier separately, observing to place the product of the unit's figure under the figure you multiply by ; if the product be 10 or more, place the unit's figure down, and carry the tens.

5. Add these products into one sum, which will be the whole product required ; as follows :—

Multiply 4567 by 325

$$\begin{array}{r}
 4567 \\
 \times 325 \\
 \hline
 22835 \\
 9134 \\
 13701 \\
 \hline
 1484275 \\
 \hline
 \end{array}
 =
 \begin{array}{r}
 5 \text{ times } 4567 \\
 20 \text{ times } 4567 \\
 300 \text{ times } 4567 \\
 \hline
 325 \text{ times } 4567 \\
 \hline
 \end{array}$$

Multiply 365789 by 2005.

$$\begin{array}{r}
 365789 \\
 \times 2005 \\
 \hline
 1828945 \\
 73157800 \\
 \hline
 733406945 \text{ Product.} \\
 \hline
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. Multiply 4567896 by 3.
2. Multiply 3672817 by 4.
3. Multiply 3246129 by 5.

4. Multiply 7800406 by 6.
5. Multiply 87650504 by 7.
6. Multiply 87620919 by 8.
7. Multiply 36728514 by 9.
8. Multiply the 9 digits by 11.
9. Multiply 987654321 by 12.
10. Multiply 367846 by 13.

367846	or	367846
13		1103538
1103538		4781998
367846		
4781998		

NOTE.—This is done by multiplying by the three, and placing the product one figure to the right.

- | | |
|--|---------------------------|
| 11. Multiply 876214 by 14. | <i>Ans.</i> 12266996. |
| 12. Multiply 876298 by 15. | <i>Ans.</i> 13144470. |
| 13. Multiply 23567392 by 21. | <i>Ans.</i> 494915232. |
| 14. Multiply 47187413 by 24. | <i>Ans.</i> 1132497912. |
| 15. Multiply 2097367 by 72. | <i>Ans.</i> 151010424. |
| 16. Multiply 23577367 by 264. | <i>Ans.</i> 6224424888. |
| 17. Multiply 215357467 by 1486. | <i>Ans.</i> 320021195962. |
| 18. Multiply 37157437 by 7486. | <i>Ans.</i> 278160573382. |
| 19. What is the product of 36784 by 369? | <i>Ans.</i> 13573296. |

NOTE.—The following method of proof, as in Addition, is not infallible, though very convenient.—See the first note in Addition.

Cast the nines out of the multiplier and multiplicand, as in Addition, and multiply the two excesses or remainders together; then if the excess of the nines in this product, be equal to the excess of the nines in the total product, the work is said to be right.

Multiply	4577
by	58
	36616
	22885
	265466

	2	
5	—	4
	2	

CASE II.—*When there are ciphers on the right of the multiplicand or the multiplier, or both.*

RULE.—Multiply the figures only, and place as many ciphers to the right of the product as there are ciphers in both the factors ; as follows—

Multiply 46895 by 300.

46895		46895
300	or	300
14068500		14068500

Multiply 785400 by 5200.

785400		785400
5200	or	5200
15708		157080000
39270		3927000
4084080000		4084080000

NOTE.—Multiplying any number by 10, is only adding a cipher to the right of it ; multiplying by 100 is adding two ciphers ; and so on.

EXAMPLES FOR PRACTICE.

1. Multiply 471000 by 40700. *Ans.* 19169700000.
2. Multiply 507000 by 30500. *Ans.* 15463500000.
3. What is the sum, difference, and product of 361 and 54 ? *Ans.* 415 *sum*, 307 *dif.*, 19494 *product*.
4. Required the sum, difference, and product of 321 and 123. *Ans.* 444 *sum*, 198 *dif.*, 39483 *product*.
5. Bought 125 pieces of cloth, each 25 yards : how many yards in the whole ? *Ans.* 3125 *yards*.
6. If one quarter of wheat weigh 37 stones, what will be the weight of 365 quarters ? *Ans.* 13505 *stones*.

CASE III.—*When the multiplier is the product of two or more numbers.*

RULE.—Multiply the multiplicand by one of the component parts or numbers, and this product by the next component part or number, and so on for the rest, as follows :—

Multiply 31416 by 56.

Here the multiplier 56 is composed of the two component parts 8 and 7, that is 8 times 7 make 56.

$$\begin{array}{r} 31416 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 251328 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1759296 \\ \hline \end{array}$$

or by Case 2: $\begin{array}{r} 31416 \\ 56 \\ \hline \end{array}$

$$\begin{array}{r} 188496 \\ 157080 \\ \hline \end{array}$$

$$\begin{array}{r} 1759296 \\ \hline \end{array}$$

Multiply 37846 by 112.

Here 112 is composed of 4, 4, and 7, or $4 \times 4 \times 7 = 112$

$$\begin{array}{r} 37846 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 151384 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 605536 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4238752 \\ \hline \end{array}$$

Or by Case 2: $\begin{array}{r} 37846 \\ 112 \\ \hline \end{array}$

$$\begin{array}{r} 75692 \\ 37846 \\ 37846 \\ \hline \end{array}$$

$$\begin{array}{r} 4238752 \\ \hline \end{array}$$

Multiply 4824 by 1728.

Here $12 \times 12 \times 12 = 1728$.

$$\begin{array}{r} 4824 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 57888 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 694656 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 8335872 \\ \hline \end{array}$$

Or by Case 2: $\begin{array}{r} 4824 \\ 1728 \\ \hline \end{array}$

$$\begin{array}{r} 38592 \\ 9648 \\ 33768 \\ 4824 \\ \hline \end{array}$$

$$\begin{array}{r} 8335872 \\ \hline \end{array}$$

EXAMPLES FOR PRACTICE.

1. Multiply 392716 by 36. *Ans.* 14137776.
2. Multiply 945698 by 45. *Ans.* 42556410.
3. Multiply 768222 by 56. *Ans.* 43020432.
4. Multiply 3564184 by 96. *Ans.* 342161664.

5. Multiply 30917860 by 132. *Ans.* 4081157520.
 6. Multiply 246913578 by 14400. *Ans.* 355555523200.
 7. Multiply the 9 digits by 1728. *Ans.* 21333331392.
 8. Multiply the 9 digits by the 9 digits inverted ; that is, 123456789 by 987654321. *Ans.* 121932631112635269.
 9. What is the difference between twice fifty-six and twice six and fifty? *Ans.* 50.
 12. Multiply twelve thousand, twelve hundred, and twelve, by thirteen thousand, thirteen hundred, and thirteen. *Ans.* 189103356.

SIMPLE DIVISION.

DIVISION is the method of finding how often one number is contained in another.

The number to be divided is called the **dividend**.

The number by which the dividend is to be divided, is called the **divisor**.

The number of times the divisor is contained in the dividend, is called the **quotient**.

Divide 36 by 4.

Here 36 contains 4 nine times.

$$\begin{array}{r} 4 \overline{)36} \\ \underline{9} \\ \hline \end{array}$$

Hence 36 is called the dividend,

4 ——— the divisor,

and 9 ——— the quotient.

CASE I.—*When the divisor does not exceed 9.*

RULE.—1. Observe how often the divisor is contained in the first figure of the dividend, and place this number, which is called the **quotient figure**, under the first figure of the dividend.

2. Carry 10 for each unit that remains to the next figure of the dividend, and find how often the divisor is contained in this sum, which place down, and so on, till you have made use of all the figures in the dividend.

3. If the first figure in the dividend be less than the divisor, take the first and second figures, and proceed as above.

4. Proof by Multiplication.

EXAMPLES.

1. Divide 75678 by 3.

3)75678

25226

3

75678 Proof

2. Divide 367846 by 5.

5)367846

73569 + 1 remains.

5

367846 Proof.

In the 1st example, the 7 contains the 3 twice and one over, put down the 2 under the first figure of the dividend; for the 1 which remains carry 10 to the next figure 5, and the sum 15 being divided by 3, the quotient figure will be 5, which put down, and so on to the end.

In the 2nd example, the first figure 3 in the dividend does not contain the divisor 5; consequently take the 3 and 6, and find how many 5's there are in 36: here we find it is contained 7 times and 1 over; place this 7 under the 6, and for the 1 add 10 to the next figure 7, which will make 17; then how many 5's in 17, three times and 2 remains; carry 20 to the next figure 8, and so on to the end of the division.

In this example we have 1 remaining, which in the proof must be added to the product of the first figure.

EXAMPLES FOR PRACTICE.

1. Divide 4567894 by 2.
2. Divide 5673281 by 3.
3. Divide 893246 by 4.
4. Divide 67897628 by 5, by 6, by 7, and by 12.
5. In 567840 shillings, how many crowns?
6. If a cipher be placed to the right of any given number, and this product divided by 2, the quotient will be five times the given number. Query the reason.

CASE II.—*When the divisor consists of two or more figures.*

RULE.—Find how many times the divisor is contained in the same number of figures of the dividend, as in example 1st. But if the same number of figures in the dividend be less than the divisor, take one more, as in example 2nd; and place this figure in the quotient, and multiply the divisor by it; place the product under the figures taken in the dividend, from which subtract this product; to the remainder bring

Down the next figure in the dividend, and place it to the right of the remainder; then find how often the divisor is contained in this number, and so on.

EXAMPLE 1.

Divide 367845 by 23.

$$\begin{array}{r}
 23 \overline{) 367845} \quad (15993 \\
 \underline{23} \qquad \underline{23} \\
 137 \quad 47979 \\
 \underline{115} \quad 31986 \\
 \qquad \qquad \underline{6} \\
 228 \quad \underline{\qquad} \\
 207 \quad 367845 \text{ Proof.} \\
 \underline{\qquad} \quad \underline{\qquad} \\
 214 \\
 207 \\
 \underline{\qquad} \\
 75 \\
 69 \\
 \underline{\qquad} \\
 6 \text{ Remainder.}
 \end{array}$$

EXAMPLE 2.

Divide 1461856 by 23.

$$\begin{array}{r}
 23 \overline{) 1461856} \quad (63558 \\
 \underline{138} \qquad \underline{23} \\
 81 \quad 190674 \\
 \underline{69} \quad 127116 \\
 \qquad \qquad \underline{22} \\
 128 \quad \underline{\qquad} \\
 115 \quad 1461856 \text{ Prf.} \\
 \underline{\qquad} \quad \underline{\qquad} \\
 135 \\
 115 \\
 \underline{\qquad} \\
 206 \\
 184 \\
 \underline{\qquad} \\
 22 \text{ Remainder.}
 \end{array}$$

Divide 576848 by 23.

$$\begin{array}{r}
 23 \overline{) 576848} \quad (25080 \\
 \underline{46} \qquad \underline{23} \\
 116 \quad 75240 \\
 \underline{115} \quad 50160 \\
 \qquad \qquad \underline{8} \\
 184 \quad \underline{\qquad} \\
 184 \quad 576848 \text{ Proof.} \\
 \underline{\qquad} \quad \underline{\qquad} \\
 8 \text{ Remainder.}
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. Divide 321768430 by 17. *Ans.* 18927554 $\frac{12}{17}$.
2. Divide 321147368 by 27. *Ans.* 11894346 $\frac{6}{27}$.
3. Divide 140637301 by 108. *Ans.* 1302197 $\frac{25}{108}$.
4. Divide 3405657254 by 345. *Ans.* 9871470 $\frac{104}{345}$.
5. Divide 5713070049 by 678. *Ans.* 8426357 $\frac{3}{678}$.
6. Divide 952107193214 by 210472. *Ans.* 1672940 and 165534 rem.

NOTE.—This being one of the most difficult of the four leading rules, the pupil ought to practise it with small divisors, as 3, 4, 5, 6, &c., till he thoroughly understands the nature of it ; proving each question by multiplication.

CASE III.—*When the divisor is a composite number, that is, composed of two or more component parts.*

RULE.—1. Divide the dividend by one of the component parts, and this quotient by the next component part, and so on, for the quotient required.

2. If there be a remainder after dividing by the first component part, place it down with a plus (+) before it, as in example 1.

3. If there be a remainder after dividing by the second component part, multiply this remainder by the previous divisor, to which product add the first remainder, for the true remainder.

4. If there be a remainder after the third division, multiply this remainder by the two preceding divisors, to which product add the last true remainder, and so on for any number of divisions.

1. Divide 76281 by 24.

$$\begin{array}{r} 6)76281 \\ \hline 4)12713 + 3 \\ \hline \end{array}$$

3178 quotient $1 \times 6 + 3 = 9$ remainder.

2. Divide 1236917 by 112.

$$\begin{array}{r} 7)1236917 \\ \hline 4)176702 + 3 \text{ rem.} \\ \hline \end{array}$$

$$4)44175 + 2 \times 7 + 3 = 17 \text{ rem.}$$

11043 quotient. $3 \times 4 \times 7 + 17 = 101$ rem.

EXAMPLES FOR PRACTICE.

- | | |
|-------------------------|-------------------------------|
| 1. Divide 37649 by 16. | <i>Ans. 2353 and 1 rem.</i> |
| 2. Divide 89462 by 18. | <i>Ans. 4970 and 2 rem.</i> |
| 3. Divide 783216 by 25. | <i>Ans. 31328 and 16 rem.</i> |
| 4. Divide 821234 by 35. | <i>Ans. 23463 and 29 rem.</i> |
| 5. Divide 712346 by 96. | <i>Ans. 7420 and 26 rem.</i> |

7. Divide 371416 by 132. *Ans.* 2813 and 1003 rem.
 6. Divide 846724 by 1728. *Ans.* 490 and 4 rem.
 8. Divide 746923 by 1120. *Ans.* 666 and 100 rem.
 9. In 360480 pence, how many pounds? or divide 360480 by 240. *Ans.* 1502.

CASE IV.—*When the divisor and dividend contain ciphers.*

RULE.—1. Cut off an equal number of ciphers from each before you begin the division, and proceed as in Case 2.

2. When the division is finished, the ciphers pointed off from the dividend must be placed to the right of the remainder, if any, for the true remainder.

1. Divide 456800 by 2400.

$$24,00 \quad 4)4568,00$$

$$6)1142$$

$$190 \text{ quotient } 2 \times 4 = 8 ; 800 \text{ rem.}$$

- 2 Divide 4762000 by 6500.

$$65,00)47620,00(732 \text{ quotient.}$$

$$455$$

$$212$$

$$195$$

$$170$$

$$130$$

4000 remainder.

3. Divide 783613 by 20.

$$2,0)78361,3$$

39180 quotient, 13 remains.

EXAMPLES FOR PRACTICE.

1. Divide 7846930 by 20. *Ans.* 392346½
 2. Divide 7380000 by 23000. *Ans.* 320 and 20000 rem.
 3. Divide 29628750000 by 35000. *Ans.* 846535 and 25000 rem.
 4. Divide 1022000000 by 770000. *Ans.* 1327 and 210000 rem.

5. Required the sum, difference, product, and quotient of 378 and 21. *Ans.* 399 *sum*, 357 *dif.* 7938 *prod.* 18 *quot.*

6. Add 4 times 320 to 1842, subtract 365 from the sum, and divide the remainder by 27. Query the quotient.

Ans. 102 and 3 *rem.*

7. In 36849 shillings, how many pounds? *Ans.* 1842*l.* 9*s.*

8. What number divided by 125 will have 365 for the quotient? *Ans.* 45625.

9. What number divided by 625 will have 7854 for a quotient, and leave a remainder of 125? *Ans.* 4938875.

10. What number will be 24 short of containing 4840 just 125 times? *Ans.* 604976.

CASE V.—To divide by a mixed number, as $2\frac{1}{3}$, $3\frac{2}{5}$, &c.

RULE.—Multiply the divisor and dividend by the lower figure of the fractional part, as in the following examples:—

1. Divide 46970 by $2\frac{1}{3}$.

$$\begin{array}{r} 2\frac{1}{3} \quad 46970 \\ 3 \qquad \qquad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \quad 7)140910 \\ \hline \end{array}$$

20130 quotient

2. Divide 7848 by $2\frac{2}{5}$

$$\begin{array}{r} 2\frac{2}{5} \quad 7848 \\ 5 \qquad \qquad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \quad 12)39240 \\ \hline \end{array}$$

3270 quotient

EXAMPLES FOR PRACTICE.

1. Divide 345670 by $2\frac{1}{2}$.

Ans. 138268.

2. Divide 678549 by $2\frac{3}{4}$.

Ans. 254455*7*/₈.

3. Divide 345675 by $2\frac{3}{4}$.

Ans. 125700.

4. Divide 69286 by $4\frac{3}{4}$.

Ans. 14847.

5. Divide 78548 by $5\frac{3}{8}$.

Ans. 14727*3*/₄.

6. How many suits of clothes, each containing $3\frac{3}{8}$ yards, can a tailor make out of 54 yards of cloth? *Ans.* 16 *suits.*

7. Required the sum, difference, product, and quotient of 456 and 25. *Ans.* 481 *sum*, 431 *dif.*, 11400 *prod.*, $18\frac{6}{5}$ *quot.*

8. Required the sum, difference, product, and quotient of 1728 and 72. *Ans.* 1800 *sum*, 1656 *dif.* 124416 *prod.* 24 *quot.*

9. Find the sum, difference, product, and quotient, of 456 and 96. *Ans.* 552 *sum*, 360 *dif.* 43776 *prod.* $4\frac{1}{2}$ *quot.*

10. In 345679 farthings, how many pence?

Ans. 86419 *pence*, & 3 *farthings rem.*

11. In 785461 pence, how many shillings?

Ans. 65455 *shillings*, & 1 *penny rem.*

12. In 856751 shillings, how many pounds?

Ans. 42837 pounds, & 11 shillings rem

13. Required the sum, difference, product, and quotient of $\frac{1}{2}$ of 7854 and $\frac{1}{3}$ of 375.

Ans. 4052 sum, 3802 dif. 490875 prod. $31\frac{1}{2}$ quot.

14. Required the sum of $\frac{1}{2}$ of 1842, $\frac{2}{3}$ of 126, $\frac{3}{4}$ of 1760, and $\frac{4}{5}$ of 20.

SOLUTION.

$$\begin{array}{r} 2)1842 \\ \hline \end{array}$$

$$921 = \frac{1}{2} \text{ of } 1842$$

$$\begin{array}{r} 3)126 \\ \hline \end{array}$$

$$42 = \frac{1}{3} \text{ of } 126$$

$$\begin{array}{r} 2 \\ \hline \end{array}$$

$$84 = \frac{2}{3} \text{ of } 126$$

$$\begin{array}{r} 4)1760 \\ \hline \end{array}$$

$$440 = \frac{1}{4} \text{ of } 1760$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$1320 = \frac{3}{4} \text{ of } 1760$$

$$\begin{array}{r} 5)20 \\ \hline \end{array}$$

$$4 = \frac{1}{5} \text{ of } 20$$

$$\begin{array}{r} 4 \\ \hline \end{array}$$

$$16 = \frac{4}{5} \text{ of } 20$$

$$921$$

$$84$$

$$1320$$

$$16$$

$$\hline 2341 \text{ sum.}$$

Or thus :

$$\begin{array}{r} 2)1842 \\ \hline \end{array}$$

$$126$$

$$2$$

$$921$$

$$\hline$$

$$3)252$$

$$84$$

$$1760$$

$$3$$

$$4)5280$$

$$1320$$

$$20$$

$$4$$

$$5)80$$

$$16$$

$$921$$

$$84$$

$$1320$$

$$16$$

$$\hline 2341 \text{ sum.}$$

15. Required the sum, difference, product, and quotient of $\frac{4}{5}$ of 100, and $\frac{7}{8}$ of 48.

Ans. 122 sum, 38 dif. 3360 prod. and $1\frac{1}{2}$ quot.

16. Required the sum, difference, product, and quotient of $\frac{2}{3}$ of 450 and $\frac{7}{8}$ of 96.

Ans. 184 sum, 16 dif. 8400 prod. and $1\frac{4}{7}$ quot.

ARITHMETICAL TABLES.

TABLES OF MONEY.

£ or *l.* denotes pounds, *s.* shillings, and *d.* pence.

$\frac{1}{4}$ denotes one farthing, or one quarter of anything.

$\frac{1}{2}$ ——— one half-penny, or the half of anything.

$\frac{3}{4}$ ——— three farthings, or three quarters of anything.

4 farthings	make	1 Penny, <i>d.</i>
12 pennies or pence		1 Shilling, <i>s.</i>
20 shillings		1 Pound, £
2 <i>s.</i> 6 <i>d.</i>		Half-a-crown
5 <i>s.</i>		1 Crown
6 <i>s.</i> 8 <i>d.</i>		1 Noble
13 <i>s.</i> 4 <i>d.</i>		1 Mark
27 <i>s.</i>		1 Moidore
21 <i>s.</i>		1 Guinea
20 <i>s.</i>		1 Sovereign.

FARTHINGS.

$$4 = 1d.$$

$$48 = 12d. = 1s.$$

$$960 = 240d. = 20s. = £1.$$

<i>d.</i>		<i>s.</i>	<i>d.</i>		<i>d.</i>		<i>s.</i>	<i>d.</i>
12	make	1	0		72	make	6	0
20		1	8		80		6	8
24		2	0		84		7	0
30		2	6		90		7	6
36		3	0		96		8	0
40		3	4		100		8	4
48		4	0		108		9	0
50		4	2		120		10	0
60		5	0		132		11	0
70		5	10		144		12	0
<i>s.</i>		£	<i>s.</i>		<i>s.</i>		£	<i>s.</i>
20	make	1	0		110	make	5	10
30		1	10		120		6	0
40		2	0		130		6	10
50		2	10		140		7	0
60		3	0		150		7	10
70		3	10		160		8	0
80		4	0		170		8	10
90		4	10		180		9	0
100		5	0		200		10	0

TROY WEIGHT.

24 Grains (<i>gr.</i>)	make 1 Pennyweight,	marked <i>dwt.</i>
20 Pennyweights	1 Ounce	 <i>oz.</i>
12 Ounces	1 Pound	 <i>lb.</i>

By this weight, gold, silver, platina, jewels, and precious stones are weighed. Goldsmiths and jewellers have a particular class of weights for gold and precious stones, viz., the carat and grain; and for silver the pennyweight and grain.

Weight and value of English gold and silver coins:—

GOLD.											
	£	s.	d.	dwt.	gr.		£	s.	d.	dwt.	gr.
A Sovereign...	1	0	0	...	5 3	A Guinea	1	1	0	...	5 8
Half Sovereign	0	10	0	...	2 13½	Half Guinea...	0	10	6	...	2 16
SILVER.											
A Crown	0	5	0	...	19 8½	Sixpence	0	0	6	...	1 22½
Half Crown ...	0	2	6	...	9 16¼	Fourpenny Piece	0	0	4	...	1 7
Shilling	0	1	0	...	3 21	Threepenny do.	0	0	3	...	0 23¼

NOTE.—The value of gold at the Mint is £3 17s. 10½d. per ounce, and that of silver nearly 5s. The true standard for gold coin is 22 parts of fine gold and 2 parts of copper melted together; in which case, any mass, so mixed, is said to be 22 carats fine. And 11oz. 2dwt. of fine silver and 18dwt. of copper melted together, make the standard for silver coinage.

A pound troy of standard gold makes $46\frac{89}{129}$ sovereigns, or $44\frac{1}{2}$ guineas; and a pound weight of silver makes 66 shillings.

A pound troy = 5760 grains troy.

A pound avoirdupois = 7000 grains troy.

AVOIRDUPOIS WEIGHT.

16 Drams, (<i>dr.</i>)	make 1 Ounce,	marked <i>oz.</i>
16 Ounces	1 Pound,	 <i>lb.</i>
14 Pounds	1 Stone,	 <i>st.</i>
28 Pounds or 2 Stones	1 Quarter,	 <i>qr.</i>
4 Quarters or 112lb	1 Hundredweight,	<i>cwt.</i>
20 Hundredweight	1 Ton,	 <i>ton.</i>
19½ Ditto	1 Fother of Lead,	<i>fo.</i>
9 Pounds	1 Gallon of Oil,	<i>gal.</i>
2268 Pounds	1 Ton of Oil,	<i>t.</i>

By this weight, all articles are sold, except gold, silver, platina, diamonds, or other precious stones; but drugs, when sold by retail, are commonly sold by Apothecaries' Weight.

APOTHECARIES' WEIGHT.

20 Grains, (<i>gr.</i>)	make	1 Scruple,	marked	<i>sc.</i>
3 Scruples		1 Dram,		<i>dr.</i>
8 Drams		1 Ounce,		<i>oz.</i>
12 Ounces		1 Pound,		<i>lb.</i>

This weight is used by surgeons, apothecaries, &c., in compounding medicines. It differs but little from Troy Weight, excepting in its divisions.

LONG MEASURE.

3 Barleycorns, (<i>bar.</i>)	make	1 Inch,	marked	<i>in.</i>
12 Inches		1 Foot,		<i>ft.</i>
3 Feet or 36 inches		1 Yard,		<i>yd.</i>
6 Feet		1 Fathom,		<i>fath.</i>
5½ Yards		1 Statute Pole or Rod,		<i>po.</i>
7 Yards		1 Rod of fencing or draining.		
40 Poles or 220 yards		1 Furlong,		<i>fur.</i>
8 Furlongs or 1760 yards		1 Mile,		<i>mi.</i>
3 Miles		1 League,		<i>lea.</i>
69½ English miles		1 Degree on the Equator.		

4 inches is called a hand, when used in measuring the height of horses.

This measure is used to ascertain the distances, lengths, breadths, heights, depths, &c.

CLOTH MEASURE.

2½ Inches	make	1 Nail,	marked	<i>nl.</i>
4 Nails or 9 inches		1 Quarter of a yard		<i>qr.</i>
3 Quarters, or 27 inches		1 Flemish Ell		<i>Fl. E.</i>
4 Quarters, or 36 inches		1 Yard		<i>yd.</i>
5 Quarters		1 English Ell		<i>E. E.</i>
6 Quarters		1 French Ell		<i>Fr. E.</i>

The yard is the regular standard measure in England for cloth, stuffs, &c. It is supposed that the yard was settled by Henry I. from the length of his own arm.

LAND MEASURE.

40 Square Poles or Perches (<i>p.</i>)	make	1 Rood	<i>r.</i>
4 ——— Roods		1 Acre	<i>a.</i>
4840 ——— Yards		1 Acre	<i>a.</i>
10 ——— Chains		1 Acre	<i>a.</i>

Land is usually measured by a chain, 22 yards in length; it contains 100 links, each link being 7.92 inches.

A piece of land 10 chains or 220 yards long, and 1 chain or 22 yards broad is one acre.

WINE MEASURE.

The old wine gallon contained 231 cubic inches; but the content of the new imperial standard gallon, as fixed by Act of Parliament in 1824, contains 277.274 cubic inches; which is now the standard measure of capacity in the United Kingdom, for wine, spirits, ale, beer, and all other liquids.

* 4 Gills, (<i>gil.</i>)	make 1 pint,	marked	<i>pt.</i>
2 Pints	 1 Quart,		<i>qt.</i>
4 Quarts	 1 Gallon,		<i>gal.</i>
42 Gallons	 1 Tierce,		<i>tier.</i>
63 Gallons	 1 Hogshead,		<i>hhd.</i>
84 Gallons	 1 Puncheon,		<i>pun.</i>
2 Hogsheads or 126 gallons	1 Pipe or butt,	<i>pipe</i> or <i>B.</i>	
2 Pipes or 252 gallons	1 Tun		<i>tun.</i>
+ { 10 Gallons	 1 Anker of Brandy.		
18 Gallons	 1 Runlet.		

ALE AND BEER MEASURE.

2 Pints (<i>pt.</i>)	make 1 Quart,	marked	<i>qt.</i>
4 Quarts,	 1 Gallon,		<i>gal.</i>
9 Gallons,	 1 Firkin of Beer		<i>fir.</i>
2 Firkins or 18 Gallons,	 1 Kilderkin,		<i>kil.</i>
2 Kilderkins or 36 Gallons,	 1 Barrel		<i>bar.</i>
1½ Barrels or 54 Gallons	 1 Hogshead		<i>hhd.</i>
2 Barrels or 72 Gallons,	 1 Puncheon		<i>pun.</i>
2 Hogsheads	 1 Butt		<i>butt.</i>
2 Butts or 4 hhds.	 1 Tun		<i>tun.</i>

NOTE.—Ale is now reckoned at 9 gallons to the firkin; each gallon contains 277.274 cubic inches, which is called the imperial gallon, as settled by Act of Parliament. The old ale gallon contained 282 cubic inches.

DRY MEASURE.

Dry Measure is used in measuring all dry commodities, as wheat, barley, oats, beans, &c. Fruit, roots, salt, coals, &c. but these last commodities are now generally sold by weight, —Avoirdupois.

* In the North of England, a gill is half a pint.

† The cask called an anker, usually holds from 9 to 10 gallons; and the runlet from 18 to 20 gallons.

DRY MEASURE.

2 Pints (<i>pt.</i>)	make	1 Quart	marked	<i>qt.</i>
2 Quarts, or 4 pints		1 Pottle		<i>pot.</i>
2 Pottles, or 4 quarts		1 Gallon		<i>gal.</i>
2 Gallons, or 8 quarts		1 Peck		<i>pe.</i>
4 Pecks, or 8 gallons		1 Bushel		<i>bu.</i>
2 Bushels, or 8 pecks		1 Strike		<i>st.</i>
3 Bushels, (in some parts)		1 Load		<i>ld.</i>
4 Bushels or 2 strikes		1 Sack or Comb		<i>sa.</i>
2 Sacks or 8 bushels		1 Quarter		<i>qr.</i>
5 Quarters or 40 bushels		1 Wey		<i>wey.</i>
2 Weys, or 10 quarters		1 Last		<i>last.</i>
36 Bushels		1 Chaldron of Coals.		

Coals are now sold by weight, Avoirdupois, as per Act of Parliament, which took place January 1st, 1836.

A gallon of corn contains 277.274 cubic inches. The old standard or Winchester bushel, is $18\frac{1}{2}$ inches in diameter, and 8 inches deep; and contains 2150.42 cubic inches.

The imperial or legal bushel contains 2218.198 cubic inches.

The old corn gallon contained $268\frac{1}{2}$ cubic inches.

SQUARE MEASURE.

144 Square Inches	make	1 Square Foot	marked	<i>ft.</i>
9 Square Feet		1 Square Yard		<i>yd.</i>
7 Square Yards or 63 feet		1 Rood of Walling		<i>rd.</i>
30 $\frac{1}{4}$ Square Yards		1 Square Pole		<i>po.</i>
40 Square Poles or Perches		1 Rood		<i>rd.</i>
4 Square Roods, or 4840 square yards	}	1 Acre		<i>ac.</i>

N.B. By this measure all superficies are computed.

CUBIC OR SOLID MEASURE.

1728 Cubic Inches	make	1 Cubic Foot.	
27 Cubic Feet		1 Cubic Yard.	
40 Feet of rough Timber or	} ...	1 Load or Tun.	
50 Feet of hewn Timber...			
42 Cubic Feet		1 Tun of Shipping.	

By this measure all solid bodies are computed.

TIME.

60 Seconds (<i>sec. or "</i>)	make	1 Minute, marked	<i>m. or '</i>
60 Minutes		1 Hour	<i>hr.</i>
24 Hours		1 Day	<i>day.</i>
7 Days		1 Week	<i>wk.</i>
4 Weeks		1 Common Month of 28 days	<i>mo.</i>
12 Calendar Months, or 13 common Months and 1 day, or 52 weeks and 1 day		1 Year or 365 days	<i>yr.</i>

The tropical or solar year is commonly reckoned at $365\frac{1}{4}$ days; but its exact length is 365 days, 5 hours, 48 minutes, and 48 seconds. Some astronomers make it $45\frac{1}{4}$ seconds; being the exact period of the earth's revolution round the sun.

To know the days in each month, observe :

Thirty days hath September,
April, June, and November;
February hath twenty eight alone,
And all the rest have thirty one;
Except leap year, and then's the time,
February's days are twenty and nine.

Consequently, January contains 31 days, February 28 in the common year, and 29 in the leap-year, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31.

NOTE.—The civil year is 365 days for three successive years, and 366 every fourth year or leap-year, except when the number that denotes a complete century, as the 17th, 18th, 19th, 21st.

If any year of our Lord in the present century be divisible by 4 without any remainder, it is a leap-year: but if there be a remainder, this remainder is the number of years that has past since the leap-year.

If we consider the year to consist of $365\frac{1}{4}$ days as a medium, one day ought to be struck off the account in 129 years; the solar year being 365 days, 5 hours, 48 minutes, 48 seconds.

As year denoted originally a revolution, and was not limited to that of the sun, accordingly we find by the oldest accounts, that people have at different times expressed other revolutions by it, particularly that of the moon; and consequently that the years of some accounts are to be reckoned only months and sometimes periods, as 2, 3, or 4 months. This will assist us greatly in understanding the accounts which certain nations give of their antiquity, and perhaps also of the age of man. We read, expressly in several of the old Greek writers, that the Egyptian year, at one period, was one month, or one revolution of the moon; and we are further told that at other

periods it was 3 months or 4 months ; and it is probable that the children of Israel followed the Egyptian account of their years. For a particular account of this most interesting subject, see Dr. Hutton's Mathematical and Philosophical Dictionary.

REDUCTION.*

REDUCTION is the method of reducing quantities of one name or denomination to those of another, retaining the same value.

CASE I.—*When the quantity to be reduced is a single one.*

RULE.—All great numbers are brought into small ones by multiplying them by as many of the next less as make one of the greater : thus, pounds are brought into shillings by multiplying them by 20, because 20 shillings make one pound ; shillings are brought into pence by multiplying them by 12, because 12 pence make one shilling ; and pence are brought into farthings by multiplying them by 4, because 4 farthings make one penny. See example 1.

Also, tons are brought into cwt. by multiplying them by 20, because 20 cwt. make one ton ; cwt. into quarters by multiplying them by 4, because 4 quarters make one cwt. ; quarters into pounds by multiplying them by 28, because 28 pounds make one quarter ; and pounds into ounces by multiplying them by 16, because 16 ounces make one pound. See example 2.

EXAMPLES.

1. In £5 how many shillings, pence, and farthings ?

$$\begin{array}{r}
 \text{£} \\
 5 \\
 20 \\
 \hline
 100 \text{ shillings.} \\
 12 \\
 \hline
 1200 \text{ pence.} \\
 4 \\
 \hline
 4800 \text{ farthings.}
 \end{array}$$

* The Compound rules of Addition, Subtraction, Multiplication, and Division, being each dependent upon the rules of Reduction, the author has, though contrary to custom, deemed it necessary to place this division of the subject immediately after the simple rules.

2. In 8 tons of sugar, how many cwt., quarters, pounds, and ounces?

$$\begin{array}{r}
 \text{tons.} \\
 8 \\
 20 \\
 \hline
 160 \text{ cwt.} \\
 4 \\
 \hline
 640 \text{ qr.} \\
 28 \\
 \hline
 5120 \\
 1280 \\
 \hline
 17920 \text{ lb.} \\
 16 \\
 \hline
 107520 \\
 17920 \\
 \hline
 286720 \text{ oz.}
 \end{array}$$

3. In £24, how many shillings, pence, and farthings?

Ans. 480s. 5760d. 23040 farthings.

4. In £246, how many shillings, pence, and farthings?

Ans. 4920s. 59040d. 236160 farthings.

5. In 56 guineas, how many shillings and pence?

Ans. 1176s. 14112d.

6. In 96 moidores, how many farthings? *Ans.* 124416f.

7. In 12 tons, how many cwt., quarters, pounds, and ounces?

Ans. 240cwt. 960qr. 26880lb. 430080oz.

8. In 15 pounds of silver, how many ounces, pennyweights, and grains?

Ans. 180oz. 3600dwt. 86400gr.

9. In 64 yards, how many quarters and nails?

Ans. 256 qr. 1024 nails.

10. Reduce £25 into crowns, half-crowns, and pence?

Ans. 100 crowns, 200 half-crowns, 6000d.

CASE II.—To reduce a compound quantity.

RULE.—Multiply the highest denomination of the given compound quantity by as many of the next inferior denomination as will make one of the superior, adding to that product the given parts of the next less name.

EXAMPLES.

1. In £2 15s. 6½d., how many farthings?

$$\begin{array}{r}
 \text{£ } s. \text{ d.} \\
 2 \quad 15 \quad 6\frac{1}{2} \\
 20 \\
 \hline
 55 \text{ shillings.} \\
 12 \\
 \hline
 666 \text{ pence.} \\
 4 \\
 \hline
 2667 \text{ farthings.}
 \end{array}$$

2. Reduce 2 tons, 8 cwt., 3 qr., 14 lb., 4½ oz. into fifths of ounces.

$$\begin{array}{r}
 t. \text{ cwt. } qr. \text{ lb. } oz. \\
 2 \quad 8 \quad 3 \quad 14 \quad 4\frac{1}{2} \\
 20 \\
 \hline
 48 \text{ cwt.} \\
 4 \\
 \hline
 195 \text{ qr.} \\
 28 \\
 \hline
 5474 \text{ lb.} \\
 16 \\
 \hline
 87588 \text{ oz.} \\
 5 \\
 \hline
 437943 \text{ fifths of oz.}
 \end{array}$$

3. In £5 7s. 6d. how many pence?
- Ans.*
- 1290d.

4. In £36 12s. 8½d., how many farthings?
- Ans.*
- 35169f.

5. Reduce 12 guineas, 7 shillings and 6½ pence into half-pence and farthings?
- Ans.*
- 6229 half-pence, 12458 farthings.

6. Reduce 25 moidores, 17 shillings, and 6 pence and farthings.
- Ans.*
- 8310d. 33240 farthings.

7. In 5 tons, 7 cwt., 3 qrs. 14 lb., how many ounces?

Ans. 193312oz.

8. In 12 lb., 8 oz., 16 dwt., 14 gr., how many ounces, pennyweights, and grains?
- Ans.*
- 152oz. 3056dwt. 73358gr.

9. In 4 acres, 3 roods, and 27 perches, how many roods and perches ? *Ans.* 19r. 787 perches.

10. In 12 leagues, 2 miles, 4 furlongs, how many yards and feet ? *Ans.* 67760yds. 203280 feet.

11. In 28 days, 17 hours, 50 minutes, and 12 seconds, how many hours, minutes, and seconds ?

Ans. 689 hours, 41390 min. 2483412 seconds.

CASE III.—When small names or quantities are to be brought into greater denominations, retaining the same value.

RULE.—Divide the given number by as many of the less as make one of the greater, and the remainder, if any, will be in the same denomination as the number you divided.

Thus, to bring farthings into pence, divide the farthings by 4, and the quotient will be pence, and the remainder, if any, will be farthings. To bring pence into shillings, divide the pence by 12, the quotient will be shillings, and the remainder, if any, will be pence. To bring shillings into pounds, divide the shillings by 20, the quotient will be pounds, and the remainder, if any, will be shillings.

EXAMPLES.

1. In 2667 farthings, how many pence, shillings, and pounds ?

$$\begin{array}{r} 4 \overline{)2667} \\ \hline \end{array}$$

12)666 pence and 3 farthings remain.

2,0)5,5 shillings and 6 pence remain.

2 pound and 15 shillings remain.

The following is the method of solution :

$$\begin{array}{r} 4 \overline{)2667} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \overline{)666\frac{3}{4}} \\ \hline \end{array}$$

$$\begin{array}{r} 2,0 \overline{)5,5 \ 6\frac{3}{4}} \\ \hline \end{array}$$

£2 15 6 $\frac{3}{4}$ *Ans.*

2. In 437943 fifths of ounces, how many tons ?

5)437943

16)87588 $\frac{1}{2}$ oz.

28)5474 lb. 4 $\frac{1}{2}$ oz.

4)195 qr. 14 lb. 4 $\frac{1}{2}$ oz.

20)48 cwt. 3 qr. 14 lb. 4 $\frac{1}{2}$ oz.

2 tons, 8 cwt. 3 qr. 14 lb. 4 $\frac{1}{2}$ oz.

3. In 2880 farthings, how many pence, shillings, and pounds? *Ans.* 720*d.* 60*s.* 3*l.*

4. In 33816 farthings, how much money? *Ans.* 35*l.* 4*s.* 6*d.*

5. In 11711 $\frac{1}{2}$ pence, how much money? *Ans.* 48*l.* 15*s.* 11 $\frac{1}{2}$ *d.*

6. How many pounds troy in 46080 grains? *Ans.* 8*lb.*

7. In 10368 ounces, how many cwt. *Ans.* 5*cwt.* 3 *qr.* 4*lb.*

8. In 34560 grains, how many scruples, drams, ounces, and pounds? *Ans.* 1728*sc.* 576*dr.* 72*oz.* 6*lb.*

9. In 1760 nails, how many quarters and yards? *Ans.* 440*qrs.* 110 *yds.*

10. The distance from Manchester to Liverpool, by railway, is 2154240 inches: how many miles? *Ans.* 34 *miles.*

PRACTICAL EXAMPLES.

1 In £25. how many crowns and shillings?

Ans. 100 crowns, and 500 shillings.

2 In 80 guineas; how many shillings and pounds?

Ans. 1680 *s.* and £84.

3 In £21 10*s.* 4 $\frac{1}{2}$ *d.* how many farthings?

Ans. 20659 farthings.

4. In 25200 farthings, how many guineas.

Ans. 25 guineas.

5. In 14761 farthings, how much money?

Ans. £15. 7*s.* 6 $\frac{1}{2}$ *d.*

6. In 32400 farthings, how many moidores?

Ans. 25 moidores.

7. In 1756 quarter guineas, how many guineas, shillings, and pence? *Ans.* 439 guineas, 9219 shillings, and 110628*d.*

8. In £25. three crowns, and 6*d.* how many crowns and pence? *Ans.* 103 crowns, 6186 pence.

9. In 125 moidores, how many shillings, crowns, and pounds?
Ans. 3375s. 675 crowns, and £168. 15s.

10. In 92166 grains of silver, how many lb?

Ans. 16lb. 6 grs.

11. In 4lb 4 oz. 15 dwt. 10 gr., how many grains?

Ans. 25330 grains.

12. In 12665 gr. of gold, how many lb?

Ans. 2lb 2 oz. 7 dwt. 17 gr.

13. In 11 cwt. 2 qr. 8lb, how many lb and ounces?

Ans. 1296lb, and 20736 ounces.

14. In 5 tons, 8 cwt., 2 qr., 14lb of hay, how many stones?

Ans. 869 stones.

15. In 41477 oz., how many cwt?

Ans. 23 cwt., 0 qr., 16lb, 5 oz.

16. In 6lb, how many ounces, drams, scruples, and grains?

Ans. 72 oz., 576 drs., 1728 scr., and 34560 grs.

17. Reduce 15lb. 5 oz. 4 dr. 2 scr. 12 gr. into gr.

Ans. 89092 gr.

18. In 35854 gr. of drugs, how many lb?

Ans. 6lb 2 oz. 5 dr. 1 scr. 14 gr.

19. In 7 miles, how many yards, feet, inches, and barley-corns?

Ans. 12320 yds. 36960 ft. 443520 inches, and 1330560 bar.

20. Reduce 4 leagues, 6 furlongs, 10 poles, 4 yards, into feet and inches.

Ans. 67497 ft. and 809964 in.

21. In 2328480 barley-corns, how many miles?

Ans. 12 miles, 2 fur., or 12m. 440 yds.

22. In 25 yds. 2 qr. 3 nails, how many quarters and nails?

Ans. 102 qr. and 411 nails.

23. In 12 English ells, 2 qr., how many yards?

Ans. 15 yds. 2 qrs.

24. In 18 French ells, how many Flemish ells, yards, and inches?

Ans. 36 F. E. 27 yds. and 972 inches.

25. In 3600 inches, how many nails, quarters, and yards?

Ans. 1600 nails, 400 qr. and 100 yards.

26. Reduce 15 acres, 2 roods, 12 perches, into perches.

Ans. 2492 perches.

27. In 4361 perches, how many acres.

Ans. 27 a. 1 r. 1 p.

28. Reduce 2 tuns, 1 pipe, 1 hhd. of wine, into gallons.

Ans. 693 gallons.

29. In 2100 gallons, how many puncheons?

Ans. 25 puncheons.

30. In 4 tons, 1 pipe, 1 puncheon, how many gallons of wine ? *Ans.* 1218 gal.
31. In 12 kilderkins 4 gallons and 3 quarts, (of beer or ale,) how many pints ? *Ans.* 1766 pints.
32. In 2649 pints of ale or beer, how many hogsheads ? *Ans.* 6 hhds. 7 gal. 0 qt. 1 pint.
33. In 27 quarters, 7 bushels of wheat, how many bushels and pecks ? *Ans.* 223 bushels, and 892 pecks.
34. In 1338 pecks of oats, how many quarters ? *Ans.* 41 qr. 6 bushels, 2 pecks.
35. In 5 acres, 2 roods of land, how many square yards ? *Ans.* 26620 square yards.
36. Reduce 7 acres into square chains and yards. *Ans.* 70 sq. c., and 33880 sq. yds.
37. In 35 square yards, how many square feet and inches ? *Ans.* 315 sq. ft., and 45360 sq. in.
38. In 68040 square inches, how many square feet and yds. ? *Ans.* 472 sq. ft., 72 in., and $52\frac{1}{2}$ sq. yds.
39. In 25 cubic yards, how many cubic feet and inches ? *Ans.* 675 cu. ft., 1166400 cu. in.
40. In 1749600 cubic inches, how many cubic yards ? *Ans.* $37\frac{1}{2}$ cubic yards.
41. Reduce 7 days, 20 hours, 25 minutes, and 8 seconds, into seconds ? *Ans.* 678308 seconds.
42. In 3391540 seconds, how many weeks ? *Ans.* 5 w., 4 d., 6 h., 5 m., 40 sec.
43. How many days are there in the first six months of the year ? *Ans.* 181 days.
44. How many days from the 5th of January to the 9th of October following in a leap year ? *Ans.* 278 days.
45. How many seconds are there in the tropical or solar year ? *Ans.* 31556928 seconds.
46. How many spoons, each weighing 5 oz, can be made out of 7 lb 6 oz. of silver ? *Ans.* 18 spoons.
47. How many parcels of sugar, each 2 qr. 14 lb, are there in 4 cwt. 2 qr. 21 lb. *Ans.* $7\frac{1}{2}$ parcels.
48. How many suits of clothes, each containing 3 yards, 2 qrs., 2 nails of cloth, can a tailor make out of 45 yds. 1 qr. 1 nail ? *Ans.* $12\frac{1}{2}$ suits.
49. How many crowns, half crowns, shillings, and pence, and of each an equal number, are there in £16 5s. 1d. *Ans.* 37, and 7s. 6d. over.

50. How many paces, each $2\frac{1}{2}$ feet, must a person take to walk three miles? Ans. 6336.

51. If a wheel be 15 feet in circumference, that is, round it, how many times will it turn round in 25 miles? Ans. 8800 times.

52. If the driving wheel of a railway engine be 15 feet in circumference, and one of the carriage wheels 8 feet in circumference, how many more revolutions will one wheel make than the other in travelling 200 miles? Ans. 61600.

53. In 4 cwt. 2 qr. 15 lb. avoirdupois, how many lb troy, if a lb. avoirdupois be equal to 14 oz. 11 dwt. 16 gr. troy? Ans. 630 lb. 8 oz. 15 dwt.

54. In what time could a person count 650 millions of sovereigns; reckoning 6 days in the week, and 12 hours per day, allowing him to count 100 per minute? Ans. 28 yrs. 48 weeks, 3 days, 9 hrs. 20 m.

55. How many barley-corns will reach between London and a certain place distant 204 miles, 3 furlongs, 30 poles, 3 yards, and 2 feet? Ans. 38865816 barley-corns.

56. How many seconds from the birth of Christ to Christmas 1842, reckoning 365 days, 5 hours, 48 m. and 48 seconds for one year? Ans. 58127861376 seconds.

57. Suppose the pulse of Sir Isaac Newton, to have beat 2905929000 times during his natural life, at the rate of 65 pulsations in a minute; what was his age, reckoning 365 days 6 hours to the year? Ans. 85 years.

COMPOUND ADDITION.

By COMPOUND ADDITION, several numbers, of different denominations, are collected into one sum.

RULE.—Place the given numbers in such order that the same denominations may stand under each other. Add all the lowest denomination into one sum, which divide by as many of this denomination as make one of the next greater; set down the remainder, and carry the quotient to the next superior denomination. Proceed thus through the different denominations to the highest, which add, as in Simple Addition.

MONEY.

£	s.	d.	£	s.	d.	£	s.	d.
(1.) 34	16	4½	(2.) 74	14	9¾	(3.) 34	17	6¾ *
56	17	8	12	18	8	57	18	6½
49	12	3¾	36	15	7½	17	15	7½
74	8	6	56	18	5	84	12	7½
96	18	2½	74	12	10½	36	16	4
74	14	9	49	18	7¾	27	18	2
36	16	8	93	7	4	34	15	9¾
72	19	11¾	56	18	9	43	17	2¾
12	8	10	42	13	10½	99	19	11¾
509	13	3½						
£	s.	d.	£	s.	d.	£	s.	d.
(4.) 62	7	6½	(5.) 74	15	2¾	(6.) 74	6	8¾
66	15	7½	78	18	8½	68	17	6½
32	12	2	94	17	7½	98	18	2½
64	17	10½	64	8	6¾	67	17	6½
84	16	8¾	74	17	9½	28	12	8½
56	17	2¾	68	18	10½	56	16	10½
74	8	8	79	19	11½	12	13	11½
25	16	7½	84	16	8¾	23	19	9¾
72	17	11½	99	19	11¾	76	19	11¾
£	s.	d.	£	s.	d.	£	s.	d.
(7.) 126	16	8½	(8.) 321	17	9¾	(9.) 129	18	4½
724	17	9½	789	18	6	467	17	8½
329	12	7	567	2	4¾	232	16	7¾
164	15	8½	723	6	9½	67	17	3½
472	12	2	467	4	1	53	13	9½
84	9	9¾	234	16	7½	39	17	7½
172	11	1	46	5	5	269	15	8¾
85	6	9	123	5	10	96	8	2½
1235	14	9¾	782	18	5	74	16	9

* The author has, in the 3, 4, 5, 9, and 12th examples, introduced a small 1 next the pence for ¼ or one farthing; a small 2 for ½ or one half penny; and a small 3 for ¾ or three farthings.

TROY WEIGHT.

<i>lb. oz. dwt gr.</i>	<i>lb. oz. dwt. gr.</i>	<i>lb. oz. dwt gr.</i>
(1.) 4 11 19 23	(2.) 5 11 19 22	(3) 25 11 17 23
2 8 17 17	7 8 11 18	74 8 16 16
3 6 13 14	2 6 9 14	17 4 14 12
2 4 16 16	7 3 12 13	13 10 9 16
7 8 12 11	8 7 18 13	14 11 7 14
1 9 18 19	4 10 19 12	28 8 13 18
2 2 2 11	2 6 4 14	14 6 18 17
3 8 6 14	3 4 5 4	16 4 5 15
7 10 14 16	7 6 15 16	12 7 2 13

AVOIRDUPOIS WEIGHT.

<i>t. cwt. gr. lb.</i>	<i>t. cwt. gr. lb.</i>	<i>gr. lb. oz. dr.</i>
(1.) 8 19 3 27	(2.) 4 14 2 27	(3) 18 19 8 11
9 17 2 15	8 16 1 13	5 27 13 8
8 16 4 1½	7 16 3 19	17 12 9 14
12 13 2 14	8 14 2 11	29 11 2 6
13 14 3 16	2 13 1 12	21 26 11 15
25 12 2 12	4 12 2 4	36 17 8 15
15 13 1 18	6 10 1 21	12 12 12 12
12 2 0 9	4 9 0 19	23 23 6 7
28 4 3 18	9 11 3 28	84 15 12 2

APOTHECARIES' WEIGHT.

<i>lb. oz. dr. sc.</i>	<i>lb. oz. dr. sc.</i>	<i>lb. oz. dr. sc.</i>
(1.) 18 11 7 2	(2.) 21 11 7 2	(3) 29 11 7 1
21 10 6 1	28 10 6 1	56 9 5 2
74 8 4 2	51 9 5 2	32 8 2 1
36 2 3 1	34 4 2 1	43 7 1 2
72 10 6 1	64 3 1 2	56 10 7 1
14 9 5 2	72 2 6 1	12 8 5 2
15 2 7 1	12 8 5 1	23 1 4 2
25 8 4 2	23 2 6 2	36 9 7 1
99 9 7 1	33 9 4 1	45 11 6 1
89 7 6 2	27 7 5 2	56 0 5 2

LONG MEASURE.

	<i>m.</i>	<i>fur.</i>	<i>p.</i>	<i>yds.</i>		<i>yds.</i>	<i>ft.</i>	<i>in.</i>	<i>bar</i>		<i>lea.</i>	<i>m.</i>	<i>fur.</i>	<i>p.</i>
(1.)	25	7	39	4 $\frac{1}{2}$	(2.)	27	2	11	2	(3)	27	2	7	39
	27	6	21	3 $\frac{1}{2}$		15	1	9	1		56	1	6	31
	36	5	25	4		25	5	10	2		29	1	5	14
	47	6	17	3		44	2	3	2		38	1	2	25
	83	2	24	5		66	1	7	2		74	1	7	24
	76	4	34	2		12	4	6	1		86	2	6	26
	29	3	36	4		26	3	3	2		92	1	5	17
	56	4	24	3		58	2	1	2		88	2	5	14
	74	6	25	5		35	1	8	1		16	2	4	25

CLOTH MEASURE.

	<i>yds.</i>	<i>qrs.</i>	<i>n.</i>		<i>Fl.E.</i>	<i>qr.</i>	<i>n.</i>		<i>Fr.E.</i>	<i>qr.</i>	<i>n.</i>		<i>Fr.E.</i>	<i>E.</i>	<i>E.</i>	<i>yds.</i>
(1.)	84	3	3	(2)	27	2	3	(3)	27	5	3	(4)	25	1	1	
	46	2	3		13	1	2		29	4	2		56	1	1	
	37	3	2		25	1	3		91	3	1		84	0	1	
	43	1	1		42	2	2		48	4	2		92	1	1	
	92	3	3		88	1	2		52	3	3		27	1	1	
	88	2	2		35	2	2		61	1	2		34	1	0 $\frac{1}{2}$	
	75	2	3		46	1	3		73	2	3		29	0 $\frac{1}{2}$	1	
	25	1	2		55	2	2		35	3	2		22	0 $\frac{1}{4}$	0 $\frac{1}{4}$	
	78	2	3		64	1	1		46	2	1		71	1	1	
	63	3	2		75	2	3		55	3	3		57	1	1	

LAND MEASURE.

	<i>a.</i>	<i>r.</i>	<i>p.</i>	<i>yds.</i>		<i>a.</i>	<i>r.</i>	<i>p.</i>	<i>yds.</i>		<i>a.</i>	<i>r.</i>	<i>p.</i>
(1.)	24	3	39	20	(2.)	27	3	18	29	(3.)	38	3	27
	56	2	15	18		56	2	32	30		25	2	39
	74	1	25	12		17	3	14	19		18	2	17
	73	3	28	16		26	3	12	12		16	3	12
	45	2	35	30		17	1	13	16		14	2	18
	29	2	17	16		93	2	12	25		17	1	12
	45	3	2	9		17	1	20	14		15	2	13
	21	2	14	18		15	2	11	25		25	3	14
	29	3	39	27		17	3	13	17		37	1	28
	66	2	22	9		65	2	9	9		24	3	23

WINE MEASURE.

<i>t. pip. hhd.</i>	<i>hhd. gal. qt.</i>	<i>tier. gal. qt.</i>	<i>pun. gal. qt.</i>
(1.) 12 1 1	(2.) 25 62 1	(3.) 25 41 3	(4.) 25 83 3
15 1 1 $\frac{1}{2}$	36 24 3	45 39 2	48 77 2
27 0 1 $\frac{3}{4}$	78 16 2	74 18 1 $\frac{1}{2}$	19 13 3 $\frac{1}{2}$
74 1 1 $\frac{1}{2}$	98 29 2	43 26 2	71 12 2 $\frac{1}{2}$
56 0 1	73 21 1 $\frac{1}{2}$	74 40 3 $\frac{1}{4}$	64 29 3 $\frac{1}{2}$
15 1 1 $\frac{1}{2}$	57 59 3	29 14 2	17 29 1
17 1 1 $\frac{1}{4}$	28 14 2 $\frac{1}{4}$	37 12 2	48 81 2
27 1 1	56 56 2	46 16 2	56 29 3 $\frac{1}{4}$
33 0 1 $\frac{3}{4}$	74 18 1	21 1 1 $\frac{1}{4}$	72 21 2

ALE AND BEER MEASURE.

<i>hhd. gal. qt. pt.</i>	<i>hhd. bar. gal. qts.</i>	<i>butts. hhd. bar. gal.</i>
(1.) 36 47 3 1	(2.) 22 1 33 3	(3.) 25 1 1 53
29 23 0 0 $\frac{1}{2}$	71 1 $\frac{1}{2}$ 25 2	27 1 1 $\frac{1}{4}$ 53
74 36 2 1 $\frac{1}{4}$	27 1 17 1 $\frac{1}{2}$	57 1 1 $\frac{1}{4}$ 14
26 45 2 1 $\frac{1}{4}$	36 1 $\frac{1}{4}$ 12 2 $\frac{1}{4}$	25 1 1 36
27 27 1 2 $\frac{1}{4}$	74 1 13 1	32 1 0 44
32 29 2 1	56 3 $\frac{3}{4}$ 2 1 $\frac{1}{2}$	17 0 1 15
74 50 3 1 $\frac{1}{2}$	25 1 11 2	24 1 0 52
56 46 1 1	17 2 13 1	45 1 1 17
74 21 2 0 $\frac{1}{2}$	21 1 25 1 $\frac{3}{4}$	19 1 0 13
63 49 2 0 $\frac{1}{4}$	33 2 32 0 $\frac{1}{4}$	35 0 1 29

DRY MEASURE.

<i>lasts qr. bush.</i>	<i>qr. bush. p.</i>	<i>qr. loads bush.</i>	<i>lasts qr. p.</i>
(1.) 36 9 7	(2.) 36 7 3	(3.) 36 1 2	(4.) 96 2 31
57 8 3	74 6 2	57 2 2	17 2 25
29 7 4	86 5 1	48 1 1	94 3 13
56 3 2	49 4 3	74 2 2	73 2 25
45 4 7	72 3 2	89 1 1	84 8 27
36 5 6	81 2 1	74 2 2	39 3 28
42 4 3	74 3 2	34 1 1	74 7 27
72 7 7	81 6 3	76 1 2	56 4 29
86 2 5	99 5 2	89 1 2	72 5 22
77 6 2	53 4 1	63 2 1	55 2 31

COMPOUND ADDITION.

SQUARE MEASURE.

<i>yds.</i>	<i>ft.</i>	<i>in.</i>	<i>poles.</i>	<i>yds.</i>	<i>ft.</i>	<i>a.</i>	<i>r.</i>	<i>p.</i>	<i>yds.</i>
(1.) 21	8	143	(2.) 25	29	8	(3.) 26	3	39	28
37	8	142	56	28	4	73	2	28	17
64	7	129	72	21	5	66	1	22	13
56	6	138	64	17	3	56	2	27	16
74	5	140	56	15	4	42	1	13	15
86	8	129	44	17	3	48	2	23	17
56	7	127	66	13	7	84	1	21	19
84	6	137	84	16	2	36	2	29	12
85	7	123	91	14	3	99	3	39	29

SOLID MEASURE.

<i>yds.</i>	<i>ft.</i>	<i>in.</i>	<i>yds.</i>	<i>ft.</i>	<i>in.</i>	<i>yds.</i>	<i>ft.</i>	<i>in.</i>
(1.) 137	26	1727	(2.) 156	25	156	(3.) 47	13	256
498	25	356	942	16	174	96	26	432
345	24	436	326	17	241	83	25	724
785	26	784	344	13	269	96	17	176
986	14	128	814	23	272	84	21	134
674	18	463	867	24	172	56	17	148
329	25	476	846	19	137	56	19	823
864	18	196	129	16	118	89	25	967
864	16	473	782	13	828	77	12	172
587	23	888	631	13	235	77	15	779

TIME.

<i>hrs.</i>	<i>m.</i>	<i>s.</i>	<i>days</i>	<i>hrs.</i>	<i>m.</i>	<i>yrs.</i>	<i>days</i>	<i>hrs.</i>
(1.) 36	59	37	(2.) 84	23	52	(3.) 25	179	23
36	59	37	39	22	46	56	350	17
88	25	27	81	21	34	82	225	14
63	41	35	87	16	37	56	128	17
82	16	17	16	17	45	14	612	18
58	14	23	94	23	13	36	123	22
84	12	25	85	15	17	36	118	18
37	28	42	16	13	18	72	42	22
36	51	48	54	17	42	98	38	17

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches us to find the difference between two compound quantities.

RULE.—Place the less number under the greater, in such a manner, that the same denominations may stand under each other. Begin at the lowest denomination, and subtract the lower number from the upper one; when any of the lower denominations are greater than the upper, increase the upper number by as many as make one of the next superior denomination, from this sum take the figure in the lower line; put down the difference, and carry one to the next superior number in the lower line, and subtract as before, and so on, till you have gone through the question.

Or: When any of the lower denominations are greater than the upper, subtract the lower denomination from as many as make one of the next superior, and add the top number to the remainder; put this sum down, and carry one to the next superior denomination in the lower line, and subtract as before.

Proof as in Simple Subtraction.

EXAMPLES.

1. From £29. 12s. 6½d. take £14. 13s. 4½d.

£	s.	d.	
29	12	6½	
14	13	4½	
14	19	2½	Difference.
29	12	6½	Proof.

- 2, From 3 tons, 4 cwt. 2 qr. 15 lb. 12 oz. take 1 ton, 7 cwt. 2 qr, 18 lb. 10 oz.

t.	cwt.	qr.	lb.	oz.	
3	4	2	15	12	
1	7	2	18	10	
1	16	3	25	2	
3	4	2	15	12	Proof.

3. From 10 years take 9 years, 9 months, 9 weeks, 9 days and 9 hours.

<i>yr.</i>	<i>m.</i>	<i>wks.</i>	<i>d.</i>	<i>hrs.</i>
10	0	0	0	0
9	9	9	9	9
0	0	1	4	15
10	0	0	0	0

NOTE.—At days we have to borrow 2 weeks or 14 days, and carry 2 to the 9 weeks; at weeks we have to borrow 3 months or 12 weeks, and to carry 3 to the 9 months.

OF MONEY.

<i>£</i>	<i>s.</i>	<i>d.</i>	<i>£</i>	<i>s.</i>	<i>d.</i>	<i>£</i>	<i>s.</i>	<i>d.</i>	<i>£</i>	<i>s.</i>	<i>d.</i>
(1) 27	16	8½	(2) 86	12	4	(3) 29	14	8½	(4) 79	10	8½
13	8	4¼	29	18	2	76	16	9½	29	17	4¼

5. Gave my butler £20 to settle two accounts, one of which was £15. 7s. 6d. and the other £4. 10s. 9d. : how much had he left? *Ans.* 1s. 9d.

6. A servant man, who had £26 15s. 6d. a year; now on the 1st of June, he drew 5 guineas, on the 1st of July £2. 3s. 4d., and on the 5th of November £7. 6s. 6d. : how much money will be due to him at the year's end?

Ans. £12. 0s. 8d.

TROY WEIGHT.

<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>
(1.) 36	11	16	8	(2.) 74	1	14	21	(3.) 74	10	9	18
31	9	17	4	16	8	6	22	12	6	18	20

AVOIRDUPOIS WEIGHT.

<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	<i>tons.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>
(1.) 37	3	17	12	(2.) 29	2	4	8	(3.) 29	13	2	8
19	2	15	8	21	3	16	12	16	14	3	12

4. Two silver pints were made, one of which weighed 8 oz. 10 dwt. 21 grains, and the other 10 oz. 4 dwt. : required their difference in weight.

Ans. 1oz. 13dwt. 3gr.

5. A load of hay, with the waggon, weighed 2 tons, 12 cwt 3 qrs. : now the waggon weighed 21 cwt. 2 qrs. 14lb. : required the weight of the hay.

Ans. 1ton 11cwt. 14lb.

APOTHECARIES' WEIGHT.

	lb.	oz.	dr.	scr.	gr.		lb.	oz.	dr.	scr.	gr.		oz.	dr.	scr.	gr.
(1.)	25	1	7	1	4	(2.)	15	8	6	1	16	(3.)	11	0	2	14
	19	2	6	2	8		12	2	4	2	12		8	2	1	15

LONG MEASURE.

	m.	fur.	p.	yds.		m.	fur.	p.	yds.		yds.	ft.	in.	bar.
(1.)	25	6	12	2	(2.)	17	4	2	3	(3.)	29	2	1	1
	12	4	8	4 $\frac{1}{2}$		12	6	18	4		14	2	3	2

CLOTH MEASURE.

SOUTH BUCKS.					SOUTH BUCKS.					SOUTH BUCKS.				
	yds.	qrs.	n.	in.		E.E. yds	Fl.E. qr.		Fr. E. E.E. yds.	Fl.E.		Fr. E. E.E. yds.	Fl.E.	
(1.)	25	3	2	2	(2.)	25	1	2	1	(3.)	15	0	0	0
	17	2	3	0 $\frac{1}{4}$		17	1	2	2		12	1	1	1

LAND MEASURE.

	a.	r.	p.			a.	r.	p.			a.	r.	p.	yds.
(1.)	23	2	6		(2.)	22	1	27		(3.)	127	2	6	27
	12	3	18			19	2	17			87	3	14	30

WINE MEASURE.

	pipes	hhd.	gal.		tuns	pi.	gal.		hhd.	gal.	qt.
(1.)	4	1	42	(2.)	14	1	84	(3.)	21	25	2
	1	1	56		9	1	120		17	62	2½

ALE OR BEER MEASURE.

	kil.	gal.	qt.		hhd.	bar.	gal.		butts.	hhd.	bar.		butts.	gal.	qt.
(1.)	25	12	2	(2.)	27	1	4	(3.)	25	2	0	(4.)	27	72	2
	17	15	1		16	1	5		16	1	1		19	99	3

DRY MEASURE.

	<i>bu.</i>	<i>pecks.</i>	<i>gal.</i>		<i>qr.</i>	<i>bu.</i>	<i>p.</i>		<i>wey.</i>	<i>qr.</i>	<i>bu.</i>		<i>last.</i>	<i>qr.</i>	<i>bu.</i>
(1)	25	2	1	(2)	25	3	1	(3)	72	3	6	(4)	74	2	0
	17	2	1½		19	6	2		27	4	2		68	1	6½

SQUARE MEASURE.

	<i>yds.</i>	<i>ft.</i>	<i>in.</i>		<i>poles</i>	<i>yds.</i>	<i>ft.</i>		<i>a.</i>	<i>r.</i>	<i>p.</i>		<i>poles</i>	<i>yds.</i>	<i>ft.</i>
(1)	36	7	25	(2)	36	20	0	(3)	27	2	0½	(4)	96	4	0½
	12	7	36		27	25	8		29	1	4		29	6	8

CUBIC OR SOLID MEASURE.

	<i>yds.</i>	<i>ft.</i>	<i>in.</i>		<i>yds.</i>	<i>ft.</i>	<i>in.</i>		<i>yds.</i>	<i>ft.</i>	<i>in.</i>		<i>yds.</i>	<i>ft.</i>	<i>in.</i>
(1)	36	18	72	(2)	78	18	136	(3)	72	6	28	(4)	92	8	0
	18	25	96		28	16	728		29	19	21½		74	2	9

TIME.

	<i>h.</i>	<i>m.</i>	<i>s.</i>		<i>days</i>	<i>h.</i>	<i>m.</i>		<i>yrs.</i>	<i>wks.</i>	<i>days</i>		<i>yrs.</i>	<i>m.</i>	<i>w.</i>	<i>d.</i>
(1)	25	45	27	(2)	37	20	0	(3)	12	15	2	(4)	25	0	0	0
	18	56	28		32	21	25		7	25	10		13	13	13	13

PRACTICAL EXAMPLES.

1. Bought 17 cwt. 2 qr. 15 lb., and sold 7 cwt. 3 qr. 16 lb. how much had I left? *Ans.* 9 cwt. 2 qr. 27 lb.

2. A druggist, after disposing of 10 lb. 4 oz. 1 dr. 2 sc. 8 gr. from a cask which at first contained 15 lb.: I wish to know how much remains. *Ans.* 4lb. 7oz. 6dr. 0sc. 12gr.

3. A gentleman divided 100 acres of land between his two sons; the eldest had for his share 64 acres, 1 rood, 15 poles: required the youngest son's share. *Ans.* 35a. 2r. 25p.

4. A person set out to travel to a certain place, distant 97 miles, and after he had gone 51 miles, 3 furlongs, 2 poles, and 3 yards, what distance was he from the place? *Ans.* 45m. 4f. 37p. 2½yds.

5. A piece of silk measures 54 yards, and a piece of cloth 37 yards, 3 quarters, and 2 nails: required their difference in length. *Ans.* 16yds. 0qrs. 2n

6. A pipe of wine, which was leaky, was found to be 10 gallons, 3 quarts 1 pint deficient: required the quantity left.

Ans. 115gal. 0qts. 1pt.

7. Two porter vats, one containing 27 butts, 1 hhd. 1 bar. 37 gallons, and the other 30 butts: required their difference.

Ans. 1butt 1hhd. 0bar 35gal.

8. Two pieces of building, one measuring 220 square yards, and the other 150 yards, 7 feet, 36 inches: required their difference in area.

Ans. 69yds 1ft. 108in.

9. Two reservoirs, out of one was dug 942 cubic yards, 16 feet, 174 inches, and out of the other 793 yards, 21 feet, 258 inches: how much did the one exceed the other in size?

Ans. 148yds. 21ft. 1644in.

10. A was born at 2 o'clock in the afternoon on the 6th of April, 1837, and B on the 28th of January, at 5 o'clock in the morning, in 1842. How much does A's age exceed B's. allowing 30 days to a month.

Ans. 4yrs. 9m. 21days 15hrs.

COMPOUND MULTIPLICATION.

By COMPOUND MULTIPLICATION we find the value or amount of any given number of different denominations.

CASE I.—When the given quantity does not exceed 12.

RULE.—Multiply the price or compound quantity by the given number, beginning with the lowest denomination; divide this product by as many of this denomination as will make one of the next superior denomination; put down the remainder, and carry the quotient to the product of the next denomination, and so on, to the end; and the result will be the value or amount required.

EXAMPLES.

1. What is the value of 8 yards of cloth, at 15s. 4d. per yard?

s.	d.
15	4
	8

£6 2 8 *Ans.*

2. Required the weight of 4 hogsheads of sugar, each weighing 7 cwt. 2 qrs. 15 lb.

cwt. qr. lb.

7 2 15
4

30 2 4 *Ans.*

3. Multiply £4. 6s. 8d. by 4. *Ans.* £17. 6s. 8d.
4. Multiply £1. 6s. 3d. by 8. *Ans.* £10s. 10s.
5. Required the value of 9 yards, at 17s. 6½d. per yard. *Ans.* £7 17s. 10½d.
6. What is the value of 7 yards, at 14s. 10½d. per yard? *Ans.* £5 4s. 1½d.
7. What is the value of 9 cwt., at £2 15s. 4½d. per cwt? *Ans.* £24 18s. 4½d.
8. What is the value of 10 yards, at 9s. 8½d. per yard? *Ans.* £4 17s. 3½d.
9. What is the value of 11 yards, at 10s. 11½d. per yard? *Ans.* £6 0s. 9½d.
10. What is the value of 12 quarters of oats, at 17s. 4½d. per quarter? *Ans.* £10 8s. 6d.
11. What is the value of 12 cwt. at £1 4s. 9½d. per cwt.? *Ans.* £14 17s. 6d.
12. Multiply 4 oz. 5 dwt. 12 gr. by 6. *Ans.* 2 lb. 1 oz. 13 dwt.
13. Multiply 1 cwt. 2 qr. 8 lb. by 12. *Ans.* 18 cwt. 3 qr. 12 lb.
14. Multiply 4 lb. 3 oz. 4 dr. 2 sc. 18 gr. by 5. *Ans.* 21 lb. 5 oz. 7 dr. 2 sc. 10 gr.
15. Multiply 12 yards, 2 feet, 6 inches by 8. *Ans.* 102 yds. 2 ft.
16. Multiply 3 barrels, 4 gallons, 3 quarts by 12. *Ans.* 37 bar. 21 gal.
17. Multiply 3 quarters, 4 bushels, 2 pecks by 9. *Ans.* 32 qrs. 2 pecks.
18. Multiply 7 square feet, 120 square inches by 7. *Ans.* 6 sq. yds. 120 sq. in.
19. Multiply 5 cubic feet, 150 cubic inches by 12. *Ans.* 2 yds. 7 ft. 72 in.
20. Multiply 7 weeks, 4 days, 6 hours by 9. *Ans.* 68 weeks, 3 days, 6 hours.

CASE II.—When the given number exceeds 12, but not 144.

RULE.—Find two numbers whose product will make the given quantity; multiply the given price or compound quantity successively, or first by one of the numbers, and this product by the other number, and the last product will be the amount required.

But if two numbers or component parts cannot be found to make the given quantity, find two whose product will approximate the nearest to it, and take as many times the given price as the product of the two numbers differs from the given quantity, which add to, or subtract from the the last product, as the case may require, and the last result will be the answer, as in example 2.

EXAMPLES.

1. Required the value of 25 yards of cloth, at 17s. 6d. per yard.

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 17 \quad 6 \\
 \hline
 5 \times 5 \\
 \hline
 \pounds 4 \quad 7 \quad 6 \\
 \hline
 5
 \end{array}$$

£21 17 6 value required.

2. What is the value of 29 yards, at 12s. 6d. per yard?

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 12 \quad 6 \\
 4 \times 7 + 1 = 29
 \end{array}
 \quad \text{or thus:} \quad
 \begin{array}{r}
 \text{s.} \quad \text{d.} \\
 12 \quad 6 \\
 6 \times 5 - 1 = 29
 \end{array}$$

$$\begin{array}{r}
 2 \quad 10 \quad 0 \\
 \hline
 7
 \end{array}$$

$$\begin{array}{r}
 17 \quad 10 \quad 0 \text{ value of 28 yds.} \\
 12 \quad 6 \text{ value of 1 yd.} \\
 \hline
 \pounds 18 \quad 2 \quad 6 \text{ value required.}
 \end{array}$$

$$\begin{array}{r}
 3 \quad 15 \quad 0 \\
 \hline
 5
 \end{array}$$

$$\begin{array}{r}
 18 \quad 15 \quad 0 \text{ value of 30 yds.} \\
 12 \quad 6 \text{ value of 1 yd.} \\
 \hline
 \pounds 18 \quad 2 \quad 6 \text{ value required.}
 \end{array}$$

3. What is the value of 15 yards, at £1 2s. 6d. per yard?

Ans. £16 17s. 6d.

4. Required the value of 35 yards, at 10s. 8d. per yard.

Ans. £18 13s. 4d.

5. Required the value of 46 cwt. at £2 6s. 8d. per cwt.

Ans. £107 6s. 8d.

2. What is the value of 3456 yards, at 8s. 4½d. per yard?

$$\begin{array}{r}
 \begin{array}{r}
 \text{s.} \quad \text{d.} \\
 8 \quad 4\frac{1}{2} \times 6 \\
 10 \times 10 \times 10 \times 3 + 400 + 50 + 6 = 3456
 \end{array} \\
 \hline
 \begin{array}{r}
 4 \quad 3 \quad 9 \times 5 \\
 10
 \end{array} \\
 \hline
 \begin{array}{r}
 41 \quad 17 \quad 6 \times 4 \\
 10
 \end{array} \\
 \hline
 \begin{array}{r}
 418 \quad 15 \quad 0 \\
 3
 \end{array} \\
 \hline
 \begin{array}{r}
 1256 \quad 5 \quad 0 \text{ value of 3000 yards} \\
 167 \quad 10 \quad 0 \text{ value of 400} \\
 20 \quad 18 \quad 9 \text{ value of 50} \\
 2 \quad 10 \quad 3 \text{ value of 6}
 \end{array} \\
 \hline
 \begin{array}{r}
 \text{£}1447 \quad 4 \quad 0 \text{ value of 3456}
 \end{array}
 \end{array}$$

3. What is the value of 232 yards, at £1 2s. 6d. per yard?
Ans. £261.
4. What is the value of 378 cwt., at £2 3s. 4d. per cwt.?
Ans. £819.
5. What is the value of 369 yards, at 13s. 4d. per yard?
Ans. £246.
6. What is the value of 578 qrs. of wheat, at £3 6s. 8d. per quarter?
Ans. £1926 13s. 4d.
7. Bought 736 yards of cloth, at 12s. 4d. per yard: to what does it come?
Ans. £453 17s. 4d.
8. Find the value of 2417 yards, at 14s. 8½d. per yard?
Ans. £1774 19s. 8½d.
9. Find the value of 3271 yards of silk, at 5s. 9½d. per yard?
Ans. £947 4s. 6½d.
10. Find the value of 2715 cwt. at £1 17s. 2½d. per cwt.?
Ans. £5051 0s. 7½d.
11. Find the value of 2572 square yards of land, at 13s. 7½d. per yard.
Ans. £1752 3s. 6d.
12. Find the value of 1 acre of building ground, at £1 2s. 6d. per square yard.
Ans. £5445.
13. What sum of money will it require to relieve 425 persons; giving to each of them a crown, a half crown, a shilling, a sixpence, and a penny?
Ans. £193 0s. 5d.

COMPOUND DIVISION.

COMPOUND DIVISION is the method of finding how many times a given number is contained in another of different denominations.

RULE.—Divide the highest term of the given compound quantity or dividend by the divisor, as in Simple Division; set down the quotient under its proper name, and bring the remainder, if any, into the next inferior denomination, adding thereto the parts, of this name, in the dividend; divide this sum or number as before, and so on, and the result will be the quotient or answer.

Proof by Compound Multiplication.

EXAMPLES.

1. Divide £45 7s. 7½ by 2.

$$\begin{array}{r}
 \begin{array}{ccc}
 \text{£} & \text{s.} & \text{d.} \\
 2)45 & 7 & 7\frac{1}{2} \\
 \hline
 22 & 13 & 9\frac{1}{2} \text{ the quotient.} \\
 & & 2 \\
 \hline
 45 & 7 & 7\frac{1}{2} \text{ Proof.} \\
 \hline
 \end{array}
 \end{array}$$

- | | |
|--|--|
| 2. Divide £17 6s. 3d. by 2. | <i>Ans.</i> £8 13s. 1½d. |
| 3. Divide £25 4s. 9d. by 3. | <i>Ans.</i> £8 8s. 3d. |
| 4. Divide £17 6s. 8d. by 4. | <i>Ans.</i> £4 6s. 8d. |
| 5. Divide £136 7s. 7½d. by 5. | <i>Ans.</i> £27 5s. 6½d. |
| 6. Divide £373 5s. 4½ by 6. | <i>Ans.</i> £62 4s. 2½d. |
| 7. Divide £327 16s. 9½d. by 7. | <i>Ans.</i> £46 16s. 8½d. |
| 8. Divide £172 12s. 6d. by 8. | <i>Ans.</i> £21 11s. 6½d. |
| 9. Divide £7 17s. 10½d. by 9. | <i>Ans.</i> 17s. 6½d. |
| 10. Divide 21 lb. 4 oz. 3 dwt. 10 gr. by 10. | <i>Ans.</i> 2 lb. 1 oz. 12 dwt. 8½ gr. |
| 11. Divide 156 cwt. 3 qr. 22 lb. by 11. | <i>Ans.</i> 14 cwt. 1 qr. 2 lb. |
| 12. Divide 18 cwt. 3 qr. 13 lb. 8 oz. by 12. | <i>Ans.</i> 1 cwt. 2 qr. 8 lb. 2 oz. |

NOTE.—When the divisor exceeds 12, but is a composite number, find the component parts of it, as in Simple Division, and divide by them separately, as follows:—

13. Divide £27 17s. 6d. by 15.

$$5 \times 3 = 15$$

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 5)27 \quad 17 \quad 6 \\ \hline 3)5 \quad 11 \quad 6 \\ \hline \end{array}$$

1 17 2 Quotient or Answer.

14. Divide £236 16s. 8d. by 16. *Ans.* £14 16s. 0½d.

15. Divide £721 16s. 9d. by 18. *Ans.* £40 2s. 0½d.

16. Divide £78 17s. 4d. by 20. *Ans.* £3 18s. 10½d. ⅔

17. Divide £56 3s. 10d. by 22. *Ans.* £2 11s. 1d.

18. Divide 98 yds. 1 qr. 3 nails by 25. *Ans.* 3 yds. 3 qr. 3n.

19. Divide £1268 15s. 6d. equally among 28 persons.

Ans. £45 6s. 3¾d.

20. Divide 216 acres, 2 roods, 10 poles by 45.

Ans. 4 a. 3r. 10 p.

21. Divide £1235 18s. 6d. by 108. *Ans.* £11 8s. 10½d.

22. Divide £876 12s. 6d. by 360. *Ans.* £2 8s. 8½d.

NOTE.—When the divisor exceeds 12, but is not a composite number, divide as in Long Division.

23. Divide £325 17s. 4d. by 127.

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \quad \text{£} \quad \text{s.} \quad \text{d.} \\ 127)325 \quad 17 \quad 4(2 \quad 11 \quad 3\frac{3}{4} \quad 1\frac{1}{127} \text{ Ans.} \\ 254 \end{array}$$

$$\begin{array}{r} 71 \\ 20 \end{array}$$

$$\begin{array}{r} 127)1437(11s. \\ 127 \end{array}$$

$$\begin{array}{r} 167 \\ 127 \end{array}$$

$$\begin{array}{r} 40 \\ 12 \end{array}$$

$$\begin{array}{r} 127)484(3d. \\ 381 \end{array}$$

$$\begin{array}{r} 103 \\ 4 \end{array}$$

$$\begin{array}{r} 127)412(3 \text{ qrs.} \\ 381 \end{array}$$

$$\begin{array}{r} 31 \end{array}$$

24. Divide £107 6s. 8d. by 46. *Ans.* £2 6s. 8d.
 25. Divide £127 16s. 8d. by 59. *Ans.* £2 3s. 4d.
 26. Divide £42 9s. 4d. by 52. *Ans.* 16s. 4d.
 27. Divide 437 lb. 10 oz. 5 dwt. 20 gr. by 95.
Ans. 4 lb. 7 oz. 6 dwt. 4 gr.
 28. Divide 517 yards, 6 inches by 107.
Ans. 4 yds. 2 ft 6 in.
 29. Divide £225 1s. 8d. by 365. *Ans.* 12s. 4d.
 30. Divide 460 cwt. 3 qrs. 24 lb. by 126.
Ans. 3 cwt. 2 qr. 17 lb. 11 oz. 14 dr.
 31. Divide 367 miles, 7 furlongs, 27 poles, 4 yards, by 251.
Ans. 1 m. 3 fur. 29 p. 1 ft. 10 in.
 32. Divide 25 hhd. 42 gal. 3 quarts of wine, by 117.
Ans. 13 gal. 3 qts. & 36 rem.

NOTE.—When the divisor is a mixed number, as $3\frac{1}{2}$, $4\frac{2}{3}$, $5\frac{3}{4}$, &c., multiply the divisor and dividend by the lower number, as in Simple Division.

33. Divide £14 16s. 8d. by $4\frac{2}{3}$.

	£	s.	d.
$4\frac{2}{3}$	14	16	8
3			3
14	2)44	10	0
	7)22	5	0

3 3 $6\frac{1}{2}$ $\frac{2}{3}$ Quotient.

34. Divide £3 17s. $2\frac{1}{4}$ d. by $2\frac{1}{2}$. *Ans.* £1 10s. $10\frac{1}{4}$ d.
 35. Divide £7 6s. $8\frac{1}{4}$ d. by $3\frac{1}{2}$. *Ans.* £2 1s. 11d.
 36. Divide £3 7s. $2\frac{1}{4}$ d. by $5\frac{3}{4}$. *Ans.* 12s. 6d.
 37. Divide 5 cwt. 3 qr. 20 lb. by $5\frac{3}{4}$.
Ans. 1 cwt. 3 lb. 7 oz. 10 dr.

NORM.—Some of the following questions, though commonly placed in Compound Multiplication, properly belong to Division, on account of the fractional parts.

38. What is the value of $9\frac{1}{4}$ yards, at 12s. 8d. per yard?

	s.	d.
$\frac{1}{4}$	12	8
		9
5 14 0		
3 2	×	2
6 4		
£6 3 6		<i>Ans.</i>

	s.	d.
$\frac{1}{4}$	12	8
		9
5 14 0		
$\frac{1}{4}$	6 4	value of $\frac{1}{4}$
	3 2	do $\frac{1}{4}$
£6 3 6		<i>Ans.</i>

39. What is the value of $57\frac{7}{8}$ yards, at 12s. 6d. per yard?

$$\begin{array}{r}
 \begin{array}{c} s. \quad d. \\ \frac{7}{8} \quad 12 \quad 6 \end{array} \\
 8 \times 7 + 1\frac{7}{8} \\
 \hline
 5 \quad 0 \quad 0 \\
 7 \\
 \hline
 35 \quad 0 \quad 0 \\
 12 \quad 6 \\
 1 \quad 6\frac{3}{4} \text{ value of } \frac{7}{8} \\
 9 \quad 4\frac{1}{2} \text{ value of } \frac{6}{8} \\
 \hline
 \pounds 36 \quad 3 \quad 5\frac{1}{4} \text{ value required.}
 \end{array}$$

40. What is the value of $25\frac{1}{4}$ yards, at 12s. 6d. per yard?

Ans. £16 1s. $10\frac{1}{2}$ d.

41. What is the value of $5\frac{1}{8}$ yards, at 12s. 6d. per yard?

Ans. £3 7s. $2\frac{1}{2}$ d.

42. What is the value of $325\frac{7}{8}$ yards, at 6s. 8d. per yard?

Ans. £108 12s. 6d.

43. How many shirts, each to take $3\frac{7}{8}$ yards, can be made out of 48 yards of Irish cloth; and what will one shirt cost, if the cloth be 3s. 4d. per yard, and the charge for making 3s.?

Ans. $12\frac{3}{4}$ shirts; 15s. 11d. each.

44. How many parcels of sugar, each 2 qr. 15 lb. are contained in 2 hhds, each weighing 8 cwt. 2 qr. 15 lb.?

Ans. $27\frac{1}{4}$ parcels.

45. A pensioner has £15 19s. $4\frac{1}{2}$ d. per annum; what is his daily allowance?

Ans. $10\frac{1}{2}$ d.

46. Paid £2 10s. for the poor rate: at how much am I rated, the assessment being laid at 1s. 4d in the pound?

Ans. £37 10s.

47. Distributed £193 0s. 5d. among some poor persons, giving to each a crown, a half-crown, a shilling, a sixpence, and a penny: required the number relieved.

Ans. 425.

MENTAL CALCULATIONS.

This division of Arithmetic enables us to compute the value of any given number of articles, interest of money, &c. with greater facility than by the use of a Ready Reckoner.

TABLE.

	<i>d.</i>		<i>s.</i>	<i>d.</i>
1 Dozen Articles, at 1 each, amounts to	1		1	0
1 ———, at $1\frac{1}{4}$ ———, ——— to	$1\frac{1}{4}$		1	3
1 ———, at $1\frac{1}{2}$ ———, ——— to	$1\frac{1}{2}$		1	6
1 ———, at $1\frac{3}{4}$ ———, ——— to	$1\frac{3}{4}$		1	9
1 ———, at 2 ———, ——— to	2		2	0
1 ———, at $2\frac{1}{4}$ ———, ——— to	$2\frac{1}{4}$		2	3
1 ———, at 5 ———, ——— to	5		5	0
1 ———, at 7 ———, ——— to	7		7	0

&c., &c, that is, one dozen will cost as many shillings, as each article costs pence.

CASE. I.—When the given price is less than a shilling, and the given number does not exceed 240.

RULE.—Multiply the value of one dozen by the number of dozens contained in the given number, and the product will be the value required.

If the given number do not contain an exact number of dozens, take as many dozens as will approximate nearest to the given number, and find their value as above; add to or subtract from this value, the value of the odd numbers, as the case may require. See examples 2 and 3.

Or thus: Find the value of the given number, at one penny each, and multiply this value by the number of pence, each unit or article costs, for the value required.

EXAMPLES.

1. 36 yards, at 5d. per yard.

Here $36 = 3$ dozen; then 3 at 5s. make 15s. Ans.

Or thus: 36 at 1d. = 3s.; then $3s. \times 5 = 15s.$ Ans.

2. 38 yards, at 8d. per yard.

Here $38 = 3$ dozen and 2 over

	£	s.	d.
Then 3 at 8s. =	1	4	0
And 2 at 8d. =	0	1	4

1 5 4 Ans.

Or thus: 38 at 1d. = 3s. 2d.

Then 8 times 3s. 2d. = 25s. 4d. = £1. 5s. 4d. Ans.

3. 35 yards, at 8d. per yard.

Take 36 which make 3 dozen.

$$\begin{array}{r} \text{£. s. d.} \\ \text{then 3 at 8s} = 1 \quad 4 \quad 0 \\ \text{subtract 1 at 8d.} = \quad \quad 8 \end{array}$$

$$1 \quad 3 \quad 4 \text{ Ans.}$$

Or thus: 35 at 1d. = 2s. 11d.

then 8 times 2s. 11d. make 1l. 3s. 4d. Ans.

4. $119\frac{1}{2}$ yards, at 9d. per yard.

$$\begin{array}{r} \text{Take 120} = 10 \text{ dozen;} \\ \text{then 10 at 9s.} = \text{£. s. d.} \quad 4 \quad 10 \quad 0 \\ \text{subtract } \frac{1}{2} \text{ at 9d.} = \quad \quad 0 \quad 4\frac{1}{2} \end{array}$$

$$4 \quad 9 \quad 7\frac{1}{2} \text{ Ans.}$$

5. What is the value of 84 yards, at $5\frac{3}{4}$ d. per yard?

Here $84 = 7$ doz.; then 7 at 5s. 9d. = 2 0 3 Ans.

6. *What is the value of $146\frac{1}{2}$ yards at 7d. per yard? Or what is the interest of £146 10s. for 7 months at 5 per cent. per annum?

Here $146\frac{1}{2} = 12$ doz. and $2\frac{1}{2}$ over.

$$\begin{array}{r} \text{£. s. d.} \\ \text{then 12 at 7s.} = 84\text{s.} = 4 \quad 4 \quad 0 \\ \text{add } 2\frac{1}{2} \text{ at 7d.} = \quad \quad 1 \quad 5\frac{1}{2} \end{array}$$

$$4 \quad 5 \quad 5\frac{1}{2} \text{ Ans.}$$

7. What is the interest for 73l. 15s. for 8 months, at $2\frac{1}{2}$ per cent., per annum?

Here $2\frac{1}{2}$ per cent for 8 months is the same as 5 per cent. for 4 months; and 73l. 15s. = $73\frac{3}{4}$

Here $73\frac{3}{4} = 6$ doz. and $1\frac{3}{4}$ over.

$$\begin{array}{r} \text{£. s. d.} \\ \text{then 6 at 4s.} = 1 \quad 4 \quad 0 \\ \text{add } 1\frac{3}{4} \text{ at 4d.} = \quad \quad 7 \end{array}$$

$$1 \quad 4 \quad 7 \text{ Interest Ans.}$$

8. What is the value of 12 cwt. 2 qr. 14 lb. of hay, at $7\frac{1}{2}$ d. per stone?

Here 8 stones make 1 cwt, and 2 stones, 1 qr. then 12 cwt. 2 qr. 14 lb. = 101 stones.

* Interest at 5 per cent. per annum, being one penny in the £ per month; therefore, it is 7d. in the £. for 7 months.

101 make 8 doz. and 5 over.

8 at 7s. 6d. 3 0 0
add 5 at 7½d. 3 1½

3 3 1½ value required.

			Ans.	£.	s.	d.
9.	12 yards, at	5d. per yard	Ans.	0	5	0
10.	25 yards, at	7d. per yard	...	0	14	7
11.	49 yards, at	8d. per yard	...	1	12	8
12.	72 lb. of cheese, at	8½d. per lb.	...	2	11	0
13.	85 lb. of cheese, at	7½d. per lb.	...	2	13	1½
14.	97 yards, at	5d. per yard	...	2	0	5
15.	120 yards, at	7d. per yard	...	3	10	0
16.	122 stones, at	6½d. per stone	...	3	6	1
17.	144 yards, at	10¾d. per yard	...	6	9	0
18.	35½ lb. at	8d. per lb.	...	1	3	8
19.	37¾ lb. at	8d. per lb.	...	1	5	2
20.	17½ lb. at	8½d. per lb.	...	0	12	4½
21.	131¾ yards, at	5d. per yard	...	2	14	10½
22.	155½ yards, at	8d. per yard	...	5	3	10
23.	168 yards, at	10d. per yard	...	7	0	0
24.	180 yards, at	4½d. per yard	...	3	7	6
25.	192½ yards, at	5d. per yard	...	4	0	2½
26.	198 yards, at	5½d. per yard	...	4	10	9
27.	204 yards, at	5¾d. per yard	...	4	17	9
28.	215 yards, at	1s. 4d. per yard	...	14	6	8
29.	What is the interest of 37 <i>l.</i> for 4 months, at 5 per cent. per annum?		Ans.	12s.	4d.	
30.	What is the interest of 48 <i>l.</i> 10s. for 5 months, at 5 per cent. per annum?		Ans.	20s.	2½d.	
31.	What is the interest of 61 <i>l.</i> 15s. for 8 months, at 2½ per cent. per annum?		Ans.	20s.	7d.	
32.	What is the commission on 239 <i>l.</i> 19s. 11¾d. at 2½ per cent?			£5	19s.	11¾d.

CASE II.—When the given number is 240 but does not exceed 360.

RULE.—For 240, take as many £. as each article or unit costs pence; find the value of the remainder by case 1st., and the sum of these two values will be the value required.

NOTE.—The reason of this rule is obvious, because 240 pence make a pound sterling.

EXAMPLES.

1. What is the value of 264 yards, at 5d. per yard?

$$\begin{array}{r}
 264 \\
 240 \quad \text{at 5d. is} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 5 \quad 0 \quad 0 \end{array} \\
 \hline
 24 = 2 \text{ doz. at 5s.} \quad \begin{array}{r} 0 \quad 10 \quad 0 \\ 5 \quad 10 \quad 0 \end{array} \text{ Ans.}
 \end{array}$$

2. What is the value of 324 yards, at $7\frac{1}{2}$ d. per yard?

$$\begin{array}{r}
 324 \\
 240 \quad \text{at } 7\frac{1}{2}\text{d. is} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 7 \quad 10 \quad 0 \end{array} \\
 \hline
 84 = 7 \text{ doz. at 7s. 6d.} \quad \begin{array}{r} 2 \quad 12 \quad 6
 \end{array}
 \end{array}$$

$10 \quad 2 \quad 6$ Ans.

3. What is the value of $336\frac{1}{2}$ yards, at $10\frac{3}{4}$ d. per yard?

$$\begin{array}{r}
 336 \\
 240 \quad \text{at } 10\frac{3}{4}\text{d. is} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 10 \quad 15 \quad 0 \end{array} \\
 \hline
 96 = 8 \text{ doz. at 10s. 9d.} \quad \begin{array}{r} 4 \quad 6 \quad 0 \\ \frac{1}{2} \text{ yard} \quad 0 \quad 0 \quad 5\frac{1}{4} \end{array} \\
 \hline
 15 \quad 1 \quad 5\frac{1}{4} \text{ Ans.}
 \end{array}$$

4. What is the value of 242 lbs. at $4\frac{1}{2}$ d. per lb.

Ans. £4 10s. 9d.

5. What is the value of 253 yards, at $6\frac{1}{2}$ d. per yard?

Ans. £6 17s. $0\frac{1}{2}$ d.

6. What is the value of 264 yards, at $5\frac{1}{2}$ d. per yard?

Ans. £6 1s. 0d.

7. What is the value of $288\frac{1}{2}$ yards, at 8d. per yard?

Ans. £9 12s. 4d.

8. What is the value of $324\frac{1}{2}$ yards, at 7d. per yard?

Ans. £9 9s. $3\frac{1}{2}$ d.

9. Required the value of 2 tons, 2 cwt. 2 qrs. of hay, at $7\frac{1}{2}$ d. per stone.

Ans. 10l. 12s. 6d.

10. Required the value of 2 tons, 3 cwt. 3 qrs. 21 lb. of hay, at 8d. per stone.

Ans. 11l. 14s. 4d.

CASE III.—When the given number is 360 but does not exceed 480.

RULE.—Take 360 out of the given number, and call it as many pounds and half as many pounds, as each yard or unit costs pence.

Find the value of the remainder, that is what the given number exceeds 360, by the former cases.

NOTE.—The reason of this rule is obvious, 360 pence make £1 10s. = £1½.

EXAMPLES.

1. What is the value of $362\frac{1}{2}$ yards, at 8d. per yard?

$$\begin{array}{r} 362\frac{1}{2} \\ 360 \\ \hline 2\frac{1}{2} \end{array} \quad \begin{array}{l} \text{at 8d.} = \\ \text{at 8d.} = \end{array} \quad \begin{array}{r} \text{£ } s. \text{ d.} \\ 12 \quad 0 \quad 0 \\ 0 \quad 1 \quad 8 \end{array}$$

12 1 8 *Ans.*

2. If a person's income be $10\frac{1}{2}$ d. per day, what is it per annum?

$$\begin{array}{r} 365 \\ 360 \\ \hline 5 \end{array} \quad \begin{array}{l} \text{at } 10\frac{1}{2}\text{d.} \\ \text{at } 10\frac{1}{2}\text{d.} \end{array} \quad \begin{array}{r} \text{£ } s. \text{ d.} \\ 15 \quad 15 \quad 0 \\ 0 \quad 4 \quad 4\frac{1}{2} \end{array}$$

15 19 $4\frac{1}{2}$ *Ans.*

3. Required the value of $372\frac{1}{2}$ yards, at 5d. per yard.

Ans. 7*l.* 15*s.* 2½*d.*

4. Required the value of 385 yards, at 11d. per yard.

Ans. 17*l.* 12*s.* 11*d.*

5. Required the value of $398\frac{1}{2}$ yards, at 8d. per yard.

Ans. 13*l.* 5*s.* 8*d.*

6. Required the value of $455\frac{1}{2}$ yards, at 1s. 2d. per yard.

Ans. 26*l.* 11*s.* 5*d.*

7. Required the value of 2 tons, 17 cwt. 3 qrs. 7 lb at 8½d. per stone.

Ans. 16*l.* 7*s.* 7½*d.*

8. Required the value of 480 yards, at 11½d. per yard.

Ans. 23*l.* 10*s.*

CASE IV.—When the given number is 480 but does not exceed 720.

RULE.—Take 480 out of the given number, and call it as many pounds as each yard or unit costs half-pence; and find the value of the remainder by the above cases, which add to the former sum.

EXAMPLES.

1. What is the value of 492 yards, at $10\frac{1}{2}$ d. per yard?

$$\begin{array}{r} 492 \\ 480 \\ \hline 12 \end{array} \quad \begin{array}{l} \text{at } 10\frac{1}{2}\text{d. is} = \\ \text{is 1 doz. at 10s 6d.} \end{array} \quad \begin{array}{r} \text{£ } s. \text{ d.} \\ 21 \quad 0 \quad 0 \\ 0 \quad 10 \quad 6 \end{array}$$

21 10 6

2. What is the value of 510 yards of print, at 1s. $3\frac{1}{2}$ d. per yard?

$$\begin{array}{rcl}
 510 & & \text{£} \quad \text{s.} \quad \text{d.} \\
 480 & \text{at 1s. } 3\frac{1}{2}\text{d.} = & 31 \quad 0 \quad 0 \\
 \hline
 30 = 2\frac{1}{2} \text{ doz. at 15s. 6d.} & & 1 \quad 18 \quad 9 \\
 & & \hline
 & & 32 \quad 18 \quad 9
 \end{array}$$

3. What is the value of 492, at 11d. ? *Ans.* 22*l.* 11*s.*

4. What is the value of 569 yards, at 7d. per yard ?
Ans. 16*l.* 11*s.* 11*d.*

5. What is the value of $612\frac{1}{2}$ yards, at 11d. per yard ?
Ans. 28*l.* 1*s.* $5\frac{1}{2}$ *d.*

6. What is the value of 636 yards, at $7\frac{1}{2}$ d., 9d., and 1s. 2d. per yard ?

$$\text{Ans. } \left\{ \begin{array}{l} 19\text{l. } 17\text{s. } 6\text{d.} \\ 23\text{l. } 17\text{s. } 0\text{d.} \\ 37\text{l. } 2\text{s. } 0\text{d.} \end{array} \right.$$

7. What is the value of $696\frac{1}{2}$ yards, at 1s. 2d., 1s. $3\frac{1}{2}$ d., and 1s. 9d. per yard ?

$$\text{Ans. } \left\{ \begin{array}{l} 40\text{l. } 12\text{s. } 7\text{d.} \\ 44\text{l. } 19\text{s. } 7\frac{1}{2}\text{d.} \\ 60\text{l. } 18\text{s. } 10\frac{1}{2}\text{d.} \end{array} \right.$$

8. Required the value of 708 yards, at 9d., $10\frac{1}{2}$ d., $11\frac{1}{2}$ d. and 1s. $2\frac{1}{2}$ d. per yard.

$$\text{Ans. } \left\{ \begin{array}{l} 26\text{l. } 11\text{s. } 0\text{d.} \\ 30\text{l. } 19\text{s. } 6\text{d.} \\ 34\text{l. } 13\text{s. } 3\text{d.} \\ 42\text{l. } 15\text{s. } 6\text{d.} \end{array} \right.$$

9. What is the interest of £719 19*s.* $11\frac{3}{4}$ *d.* for seven months, at 5 per cent., per annum ? *Ans.* 20*l.* 19*s.* $11\frac{3}{4}$ *d.*

CASE V.—When the given number is 720, but does not exceed 960 or 1000.

RULE.—Take 720 out of the given number, and call this value three times as many pounds, as each article or unit costs pence.

Find the value of the remainder by the former rules.

EXAMPLES.

1. What is the value of 734 yards, at 5d. per yard ?

$$\begin{array}{rcl}
 734 & & \text{£} \quad \text{s.} \quad \text{d.} \\
 720 & \text{at 5d.} = & 15 \quad 0 \quad 0 \\
 \hline
 14 = 1 \text{ doz. and 2.} & & 0 \quad 5 \quad 10 \\
 & & \hline
 & & 15 \quad 5 \quad 10 \text{ Ans.}
 \end{array}$$

2. What is the value of 959 yards, at $9\frac{3}{4}$ d.

Here the shortest way will be to take it for 960, and as 960 farthings make one pound.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 960 \text{ at } 9\frac{3}{4}\text{d.} = 39 \quad 0 \quad 0 \\
 \qquad \qquad \qquad 9\frac{3}{4} \text{ subtract.} \\
 \hline
 38 \quad 19 \quad 2\frac{1}{4} \text{ Ans.}
 \end{array}$$

3. What is the value of $756\frac{1}{2}$ yards, at 4d., 5d., and 11d. per yard?

$$\text{Ans.} \left\{ \begin{array}{l} 12\text{l. } 12\text{s. } 2\text{d.} \\ 15\text{l. } 15\text{s. } 2\frac{1}{2}\text{d.} \\ 34\text{l. } 13\text{s. } 5\frac{1}{2}\text{d.} \end{array} \right.$$

4. What is the interest of £804 10s. for three, seven, and ten months, at 5 per cent?

$$\text{Ans.} \left\{ \begin{array}{l} 10\text{l. } 1\text{s. } 1\frac{1}{2}\text{d.} \\ 23\text{l. } 9\text{s. } 3\frac{1}{2}\text{d.} \\ 33\text{l. } 10\text{s. } 5\text{d.} \end{array} \right.$$

5. What is the value of 985 yards, 2 qrs., at $7\frac{1}{2}$ d. per yd?

$$\text{Ans. } £30 \text{ } 15\text{s. } 11\frac{1}{2}\text{d.}$$

CASE. VI.—For any number not exceeding ten thousand.

RULE.—Cut off the right hand figure, which divides the number by ten.

Find the value of the three figures by one of the above cases, which value multiply by 10; to this product add the value of the figure cut off.

EXAMPLES.

1. What is the value of $2916\frac{1}{2}$ yards, at $5\frac{1}{2}$ d. per yard?

$$\text{Here, cut the 6 off, then } 291 \text{ at } 5\frac{1}{2}\text{d} = \begin{array}{r} 6 \quad 13 \quad 4\frac{1}{2} \\ 10 \end{array}$$

$$6\frac{1}{2} \text{ at } 5\frac{1}{2}\text{d.} = \begin{array}{r} 66 \quad 13 \quad 9 \\ 0 \quad 2 \quad 11\frac{1}{2} \end{array}$$

$$£66 \quad 16 \quad 8\frac{1}{2}$$

$$\begin{array}{r}
 \text{Or thus: } 2916\frac{1}{2} \\
 \text{Here, three times } 960 = 2880 \quad \text{at } 5\frac{1}{2}\text{d.} = \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 66 \quad 0 \quad 0 \end{array}
 \end{array}$$

$$36\frac{1}{2} \text{ is } 3 \text{ doz. \& } \frac{1}{2} \text{ yd.} = \begin{array}{r} 0 \quad 16 \quad 8\frac{1}{2} \end{array}$$

$$\text{Ans. } 66 \quad 16 \quad 8\frac{1}{2}$$

2. What is the value of $4926\frac{1}{2}$ at 4d. 5d. 9d. and 11d.

$$\begin{array}{r} 4926\frac{1}{2} \\ 5 \text{ times } 960 = 4800 \text{ at } 4d. = 80 \quad 0 \quad 0 \end{array}$$

$$\underline{126\frac{1}{2} \text{ at } 4d.} \quad 2 \quad 2 \quad 3$$

$$\underline{82 \quad 2 \quad 3} \text{ value at } 4d.$$

$$\text{£}102 \quad 12 \quad 9\frac{1}{2} \text{ value at } 5d.$$

$$\text{£}184 \quad 15 \quad 0\frac{1}{2} \text{ value at } 9d.$$

$$\text{£}225 \quad 16 \quad 2\frac{1}{4} \text{ value at } 11d.$$

CASE VII.—*When the given price is shillings, or shillings and pence.*

RULE.—For each 20 contained in the given number, take as many pounds as each article or unit costs shillings; to which add the value of the remainder, and the sum will be the answer.

If the price contain pence, find for the pence by one of the former cases, and add it to the value found for the shillings.

EXAMPLES.

1. What is the value of 61 yards of cloth, at 9s. per yard?
Here, in 61 we have 3 times 20.

$$\begin{array}{r} \text{then } 3 \text{ score, at } \text{£}9. = 27 \quad 0 \\ 1 \text{ yard} = \quad \quad 9 \\ \hline 27 \quad 9 \end{array}$$

2. What is the value of 84 yards, at 8s. 5½d. per yard?

$$4 \text{ score, at } \text{£}8. = 32 \quad 0 \quad 0$$

$$4 \text{ yards, at } 8s. = 1 \quad 12 \quad 0$$

$$\text{By rule } 1st \text{ } 84 \text{ yards} = 7 \text{ dozen, at } 5s. 6d. = 1 \quad 18 \quad 6$$

$$\underline{\text{£}35 \quad 10 \quad 6} \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

1. What is the value of 40 yards, at 4s. per yard?
Ans. £8.
2. What is the value of 80 yards, at 7s. per yard?
Ans. £28.
3. What is the value of 119 yards, at 6s. per yard?
Ans. £35 14s.
4. What is the value of $79\frac{1}{2}$ yards, at 12s. per yard?
Ans. £47 11s.

5. What is the value of 145 yards, at 8s. per yard ?
Ans. £58.
6. What is the value of 159 yards, at 3s. per yard ?
Ans. £23 17s.
7. What is the value of $63\frac{1}{4}$ yards, at 14s. per yard ?
Ans. £44 9s.
8. What is the value of $80\frac{1}{2}$ yards, at 16s. per yard ?
Ans. £64 12s.

RULE OF THREE DIRECT.

RULE OF THREE DIRECT teaches by three numbers given to find a fourth, which shall have the same relation to the third, that the second has to the first.

RULE.—State the question by making that number which is of the same kind as the required answer, the second term, and the one which is immediately connected with it the first term, and the remaining quantity, which is of the same kind as the first, the third term.

Bring the first and third terms into the same name, and the second into the lowest denomination mentioned ; multiply the second and third terms, thus reduced, together, and divide the product by the first term. The quotient will be the answer to the question in the same denomination that the middle number was left in—which reduce to its proper terms.

NOTE.—In some cases, when the terms are of the same kind, the third term may be considered the middle term, and the first and second terms must be reduced into the same denomination, and proceed as above.—See Example second.

Also, when the first and second, or the first and third terms admit of a common divisor, they may be divided by it, which will in a many cases reduce the labour in the operation.—See examples 3 and 4.

RULE II.—The following general rule, which the author recommends, will solve all the different questions in both Direct and Inverse Proportions.

1. Consider which of the given terms is of the same kind as the answer or number required, and place it last in the proportion or stating.

2. If it appear from the nature of the question, that the answer will be greater than this number, place the greater of the other two numbers for the second term, and the remaining

one in the first term; but if less, then place the less of the remaining numbers for the second term, and the greater for the first term.

3. Bring the first and second terms into the same denomination, and the third into the lowest terms mentioned. Multiply the second and third terms, thus reduced, together, and divide the product by the first, and the quotient will be the answer to the question in the same denomination you left the third term in; which may be brought into the required denomination.

EXAMPLES.

1. Bought 3 yards 2 quarters of cloth for £2 2s.; what is the value of 23 yards 3 quarters?

<i>yds.</i>	<i>qrs.</i>	£	s.	<i>yds.</i>	<i>qrs.</i>	By the General Rule.							
As 3	2	:	2	2	::	23	3	:	2	2			
4			20	4		As 3	2	:	23	3	::	2	2
—			—	—		4			4			20	
14			42	95		—			—			—	
			95			14			95			42	
			—						42				
			210						—				
			378						190				
			—						380				
7 × 2 = 14	7)	3990							7)	3990			
		—							—				
		2)570							—				
		—							2)570				
		2,0)28,5	Ans. in shillings.						—				
		—							2,0)28,5				
		£14 5	Ans.						—				
		—							£14 5	Ans.			

NOTE.—The two following solutions belong to Examples 1 and 2 in the Rule of Three Inverse, by the above general rule:—

EXAMPLE 1.

<i>men.</i>	<i>men.</i>	<i>days.</i>
As 6	:	4
	:	3
	:	3
	:	—
6)		12
		—
		2 days, Ans.

EXAMPLE 2.

<i>yd.</i>	<i>qr.</i>	<i>yd.</i>	<i>qr.</i>	<i>yds.</i>
As 1	1	:	1	2
	:	:	:	12
	:	:	:	—
	:	:	:	4
	:	:	:	—
	:	:	:	5
	:	:	:	6
	:	:	:	12
	:	:	:	—
	:	:	:	5)72
	:	:	:	—
Ans.				14 yds. 1 3-5ths qrs.

3. Bought 7 yds. 3 qrs. of cloth, for £7 15s. What is the value of 46 yds. 2 qrs., at the same rate?

<i>yds.</i> <i>qrs.</i>	:	<i>£</i> <i>s.</i>	::	<i>yds.</i> <i>qrs.</i>
As 7 3	:	7 15	::	46 2
4		20		4
31		155		186
		186		
		930		
		1240		
		155		
		2,0)		
		31)28830		
		279		
		£46 10s.		Ans.
		93		
		93		
		0		

Or thus :

<i>yds.</i> <i>qrs.</i>	:	<i>£</i> <i>s.</i>	::	<i>yds.</i> <i>qrs.</i>
As 7 3	:	7 15	::	46 2
4		20		4
31)31		31)155		186
1		5		5
				2,0)93,0
				£46 10s.
				Ans.

Or thus :

<i>yds.</i> <i>qrs.</i>	:	<i>£</i> <i>s.</i>	::	<i>yds.</i> <i>qrs.</i>
As 7 3	:	7 15	::	46 2
4		6		4
31)31		Ans. 46 10		31)186
1				6

4. Bought 1320 lbs. of cotton, at 7½d. per lb. ; to what does it come?

$$\begin{array}{rcl} \text{lb.} & & \text{lb.} \\ \text{As } 1 & : & 7\frac{3}{8} :: 1320 \\ & & 8 \end{array}$$

$$\begin{array}{r} 59 \\ \hline 11880 \\ 6600 \end{array}$$

N.B. The middle term is left in 8ths of pence.

$$8)77880$$

$$12)9735$$

$$2,0)81,1 \ 3$$

$$\text{£}40 \ 11 \ 3 \text{ Ans.}$$

5. Bought $3\frac{3}{8}$ yards of cloth for £1 13s. 9d.; what is the value of 72 yards, at the same rate?

$$\begin{array}{rcl} \text{yds.} & & \text{£} \quad \text{s.} \quad \text{d.} \quad \text{yds.} \\ \text{As } 3\frac{3}{8} & : & 1 \ 13 \ 9 : 72 \\ & & 8 \quad 20 \quad 8 \end{array}$$

$$\begin{array}{r} 27 \\ \hline 33 \\ 12 \end{array}$$

$$405$$

$$2880$$

$$2304$$

$$9 \times 3 = 27$$

$$9)233280$$

$$3)25920$$

$$12)8640$$

$$2,0)72,0$$

$$\text{£}36 \text{ Ans.}$$

PRACTICAL EXAMPLES.

1. What is the value of 4 cwt. 2 qrs. of sugar, at £1 15s. 6d. per cwt ? Ans. £7 19s. 9d.

2. What is the value of 5 cwt. 2 qrs. 14 lbs. of sugar, at £1 15s. 6d. per cwt ? Ans. £9 19s. 8½d.

3. Bought 27 yards 3 qrs. 2 nails, at 6s. 9d. per yard; to what does it come? *Ans.* £9 8s. 1½d.

4. Bought 45 E. ells, 2 qrs. 2 nails of cloth, at 12s. 6d. per ell; to what does it come? *Ans.* £28 8s. 9d.

5. Bought 27 Flemish ells 2 qrs. of silk, at 4s. 9½d. per yard; to what does it come? *Ans.* £4 19s. 5d.

6. What is the value of 3 ends of cloth, each containing 23½ yards, at 9s. 8d. per yard? *Ans.* £34 8s. 9d.

7. What is the value of 26 French ells 5 qrs., at 8s. 6½d. per yard? *Ans.* £17 3s. 9½d.

8. What is the value of 12 qrs. 3 bushels 1 peck of wheat, at £3 12s. 6d. per quarter? *Ans.* £44 19s. 5½d.

9. Bought 3 yards of cloth for £2 6s. 6d.; what is the value of 23½ yards of the same? *Ans.* £18 8s. 1½d.

N.B. The pupil may now solve, by this rule, the questions given under the last case in Practice.

10. Bought 2 qrs. 14 lb. of soap for £1 18s. 1d.; what did it cost per lb? *Ans.* 6½d.

11. Bought 15 cwt. 2 qrs. 7 lb. for £54 9s. 4½d.; what is the value of 1 cwt.? *Ans.* £3 10s.

12. Bought 25 Flemish ells of cloth for £15 15s. 6d.? what is the value of 1 yard? *Ans.* 16s. 9¾d.

13. Bought 3 pieces of cloth, containing 72 yards 2 qrs. 2 nails, for £45 12s. 6d.? what is the value of 1 yard? *Ans.* 12s. 6¾d.

14. Bought 23 cwt. 18 lb. of sugar, for £109? what is the value of 1 cwt.? *Ans.* £4 14s. 1½d.

15. Bought 3 pieces of cloth, each containing 23 yds. 2 qrs. 2 nails, for £50 12s.; what is the value of 3¾ yards? *Ans.* £2 13s. 6½d.

16. What is the value of 23 yards 2 qrs. 2 nails of cloth, if 3 yards 3 qrs. cost £2 13s. 6½d.? *Ans.* £16 17s. 3¾d.

17. What is the value of 23 cwt. 2 qrs. 7 lb. of hay, at 7¾d. per stone? *Ans.* £6 1s. 8½d.

18. Bought 2¼ yards of cloth, for £2 17s. 6d.; what is the value of 27½ yards? *Ans.* £27 12s. 6d.

19. Bought 27½ yards of cloth for £13 16s. 3d.; what is the value of 4½ yards? *Ans.* £2 3s. 9d.

20. Bought 3 qrs. 14 lbs. of sugar for £3 1s. 3d.; what is the value of 8 cwt 3 qrs. 7½ lb. *Ans.* £30 17s. 2½d.

21. Bought 17 cwt. 2 qrs. 15 lb. of wool for £61 14s. 4½d.; what is the value of 1 qr. 21 lb. of the same? *Ans.* £1 10s. 7½d.

22. Bought 9 tons 13 cwt. 3 qrs. 19 lb. for £143 7s. 7d.; what is the value of 30 cwt.? *Ans.* £22 3s. 7½d.

23. Bought 15 cwt. for £11 1s. 9½d.; what is the value of 48 cwt. 1 qr. 25½ lb. ? *Ans.* £35 16s. 10½d.

24. Bought 7 oz. 11 dwts. of silver plate for £3 7s. 5½d.; what is the value of 2 oz. 10 dwts. 8 gr. of the same ?

Ans. £1 2s. 5½d.

25. What is the value of 37 cwt. 2 qr. 14 lb. of hay, at £4 15s. 6d. per ton ? *Ans.* £8 19s. 7½d.

26. Bought 10 hhds. of sugar, each containing 8 cwt. 3 qr. 14 lb., at 7s. 6d. per stone. To what does it come ?

Ans. £266 5s.

27. Bought 3¼ yards of cloth for £1 18s. 9d.; what is the value of 57½ yards ? *Ans.* £28 16s. 3d.

28. Bought 37 yards of silk for £6 3s. 4d.; what is the value of 1 English ell ? *Ans.* 4s. 2d.

29. Bought 3½ yards of cloth for £3 4s.; what is the value of 25 yards ? *Ans.* £22 10s.

30. Bought 79½ yards of silk for £13 8s. 9d.; what is the value of 13 yards ? *Ans.* £2 3s. 10½d.

31. Bought 357 lbs. of cotton for £6 10s. 1½d.; what is the value of 135 lbs. ? *Ans.* £2 9s. 2½d.

32. If a servant girl enter to her situation on the 5th of Nov; how much will she have due for wages this day, April 27th, at 8 guineas per annum ? *Ans.* £3 19s. 7½d.

33. My servant, who has 22 guineas per annum, entered to his situation on the 12th of July last; how much is due for wages on Christmas day following, supposing he had drawn on account 3½ guineas ? *Ans.* £6 16s. 7½d.

34. A person fails in £635 debt, and he has only £105 16s. 8d. to pay it with. How much will he pay in the pound, and what sum shall I receive for my debt of £56 18s. 6d. ?

Ans. 3s. 4d. in the pound; £9 9s. 9d. the sum.

35. A person became bankrupt, his debts amounted to £7,568 18s., and his assets, after deducting expenses, to £946 2s. 3d. How much will this person pay in the pound, and what sum shall I receive for my debt of £100 ?

Ans. 2s. 6d. in the pound; £12 10s. the sum.

36. A person became bankrupt, his debts amounted to £3,566 5s., his assets to £913 5s. Now, the different expenses in working the commission, &c. amounted to £200, which are to be paid out of the assets. What will this person pay in the pound, and what shall I receive for my debt of £70; also, what sum shall I lose by this person ?

Ans. 4s. in the pound; £14 sum received; £56 loss.

37. What is the value of 3 tons 12 cwt. 2 qr. of oil, at £34 per ton ? *Ans.* £121 14s. 6½d.

N.B. 2268 lbs. make 1 ton of oil.

38. If a gentleman's income be £488 5s. per annum ; how much may he spend per day, and lay up 100 gs. annually.

Ans. £1 1s.

39. If a gentleman's estate be worth £564 12s. per annum, and he be rated at 3s. 9d. in the pound for land tax—what is his neat income ?

Ans. £458 14s. 9d.

RULE OF THREE INVERSE.

RULE OF THREE INVERSE, or **Inverse Proportion**, is, when more requires less, or less more. More requires less, when the third term is greater than the first ; and less requires more, when the third term is less than the first.

RULE.—State the question, as in **Direct Proportion**, and reduce the numbers or quantities as therein stated ; then multiply the first and second terms together, and divide the product by the third, and the quotient will be the answer to the question in the same denomination the second term was left in. Or, by the general rule, given in the **Rule of Three Direct**.

EXAMPLES.

1. If 4 men can mow a field in three days, how many days will it take 6 men to do the same ?

$$\begin{array}{rclcl} \text{men.} & \text{days.} & & \text{men.} & \\ \text{As } 4 & : & 3 & :: & 6 \\ & & 4 & & \end{array}$$

$$\begin{array}{r} 6 \overline{) 12} \\ \underline{6} \end{array}$$

2 Days. *Ans.*

By General Rule :

$$\begin{array}{rclcl} & \text{men.} & \text{men.} & \text{days.} & \\ \text{As } 6 & : & 4 & :: & 3 \\ & & 3 & & \end{array}$$

$$\begin{array}{r} 6 \overline{) 12} \\ \underline{6} \end{array}$$

2 Days. *Ans.*

NOTE.—Here more requires less, that is, 6 men require less time to mow the grass than 4 men do ; consequently, the fourth term will be less than the second.

2. How much cotton that is $1\frac{1}{2}$ yards wide, will it take to line 12 yards of cloth, that is $1\frac{1}{2}$ yards wide ?

$$\begin{array}{rclcl} \text{yd.} & \text{qr.} & & \text{yd.} & \text{qr.} \\ \text{As } 1 & 2 & : & 12 & :: & 1 & 1 \\ & 4 & & 6 & & 4 \end{array}$$

$$\begin{array}{r} 6 \overline{) 6} \\ \underline{6} \end{array}$$

$$\begin{array}{r} 5 \overline{) 72} \\ \underline{50} \end{array}$$

$$\begin{array}{r} 6 \overline{) 6} \\ \underline{6} \end{array}$$

14 yds. $1\frac{1}{2}$ qrs. *Ans.*

NOTE.—In this question less requires more, that is, less width requires a greater length ; consequently, the fourth term will be greater than the second.

3. A room floor measures 16 feet, 6 inches, by 15 feet, 6 inches; how many yards of carpet that is 21 inches wide will cover the same?

$$\begin{array}{rclclcl} & \text{ft.} & \text{in.} & & \text{ft.} & \text{in.} & & \text{in.} \\ \text{As} & 15 & 6 & : & 16 & 6 & :: & 21 \\ & 12 & & & 12 & & & \end{array}$$

186

198

186

1188

1584

198

3)36828

7)12276

12)1753 $\frac{1}{2}$ inches.

3)146 ft. 1 $\frac{1}{2}$ in.

48 yds. 2 ft. 1 $\frac{1}{2}$ in. *Ans.*

Or by General Rule :

$$\begin{array}{rclclclclclclcl} & \text{in.} & & \text{ft.} & \text{in.} & & \text{ft.} & \text{in.} & & \text{yds.} & \text{ft.} & \text{in.} \\ \text{As} & 21 & : & 15 & 6 & :: & 16 & 6 & : & 48 & 2 & 1\frac{1}{2} \text{ } \textit{Ans.} \end{array}$$

EXAMPLES.

1. If 17 men can build a wall in 21 days; how many will do the same work in 51 days? *Ans.* 7 men.

2. If 8 men require 24 days to build a boat; how many days will it take 3 men to do the same? *Ans.* 64 days.

3. Bought 3 yards of cloth, 1 $\frac{1}{4}$ yards wide; how much stuff that is 3 quarters wide, will it take to do the same? *Ans.* 7 yards.

4. If it requires 17 yards of silk that is 2 qrs. 3 nails in width, to make a garment; how many yards of muslin that is 1 yard wide, will it take to make the same? *Ans.* 11 yds. 2 qr. 3 nails.

5. If it require 86 $\frac{1}{2}$ yards of damask, 3 qrs. 2 nails wide, to hang a bed; how many yards of calico that is 1 $\frac{1}{4}$ yards wide, will it take to line the same? *Ans.* 60 yds. 2 qrs. 3 $\frac{1}{2}$ nls.

6. Bought 12 yards of silk, 2 qrs. 3 nails in width, what will the muslin cost to line the same, at 1s. 8d. per yard, if it be $\frac{2}{3}$ yard wide? *Ans.* 18s. 4d.

7. A room measure 18 feet 6 inches in length, and 14 feet 8 inches in breadth; how many yards of carpet, 1 yard wide, will it take to cover it? *Ans.* 90 yds 1 ft. 4 in.

8. A room is 63 feet round, and 10 feet high; how many yards of paper will it take to cover the walls, the paper being 21 inches in breadth? *Ans.* 120 yards.

9. How many pounds of sugar, at $7\frac{1}{2}$ d. per lb., must be given for 12 lb. of tea, at 5s. per lb.? *Ans.* 96 lb.

10. A has 3 lb. 8 oz. of pepper, worth $13\frac{1}{2}$ d. per lb.; B has ginger worth $15\frac{1}{2}$ d. per lb.; how much ginger is equal in value to the pepper? *Ans.* 3 lb. $1\frac{1}{4}$ oz.

COMPOUND PROPORTION.

COMPOUND PROPORTION consists of 5, 7, 9, 11, 13, 15, &c. conditional terms, to find a 6th, an 8th, a 10th, a 12th, a 14th, a 16th, &c. term respectively. When 5 terms only are given to find a 6th, it is called the *Double Rule of Three*, because it may be solved by two Single Rule of Three statings.

If 5 numbers be given to find a 6th, two statings will be required; if 7 numbers be given to find an 8th, three statings will be required; if 9 to find a 10th, four statings; and so on.

RULE.—Place the statings regularly under each other, using that quantity for the middle term, which is of the same name and nature with the required term or answer.

Let all the conditional terms, or terms of supposition, be put for the first term in each stating, and their corresponding terms of demand in the third place.

NOTE.—The middle term is only used once, and is placed in the first stating, but the terms in the other statings have a due reference to it.

Examine the statings separately, and mark the conditional or first terms for divisors, when the stating is direct; but if the stating be inverse, mark the third terms.

Reduce the like terms into the same denomination, and the middle one into the lowest term mentioned therein; multiply all the marked terms together for a divisor, and the remaining terms together for a dividend; divide this dividend by the divisor, and the quotient will be the answer to the question in the same denomination you left the middle term in.

Or : Draw a line, and place all the unmarked terms above it, with the sign of multiplication between them, for a dividend, and all the marked terms below the line, with the sign of multiplication between them for a divisor.

NOTE—If two equal numbers be found, one in the divisor and one in the dividend, they may be struck out or omitted in the multiplication.

RULE II.—The following rule for the Double Rule of Three is commonly given:—

1. Let the principal cause of loss or gain, interest or decrease, action or passion, be put in the first place.

2. Let that which betokeneth time, distance of place, and the like, be put in the second place; and the remaining one in the third place.

3. Place the remaining terms under their like in the supposition.

4. If the blank fall under the third term, multiply the first and second terms for a divisor, and the other three for a dividend; but if the blank fall under the first or second terms, multiply the third and fourth terms for a divisor, and the other three for a dividend; divide one by the other, and the quotient will be the answer.

EXAMPLES.

1. If 7 men can mow 126 acres of grass in 12 days; how many acres will 16 men reap in three days?

By the rule for Compound Proportion :

$$\begin{array}{rcccl} & \text{days.} & \text{acres.} & \text{days.} & \\ \text{As} & + 12 & : 126 & :: 3 & \\ & \text{men.} & & \text{men.} & \\ & + 7 & : - & :: 16 & \end{array}$$

$$\text{Then } \frac{126 \times 3 \times 16}{12 \times 7} = \frac{6048}{84} = 72 \text{ acres.}$$

Here the two statings are direct; consequently the first terms become divisors.

By Rule 2nd.

$$\begin{array}{rcccl} & \text{men.} & \text{days.} & \text{acres.} & \\ \text{As} & 7 & : 12 & :: 126 & \\ & 16 & : 3 & :: - & \end{array}$$

Here the blank falls under the third term.

$$\text{Then } \frac{126 \times 16 \times 3}{7 \times 12} = \frac{6048}{84} = 72 \text{ acres. } \textit{Ans.}$$

2. If a man can travel 300 miles in 10 days when the day is 12 hours long; how many days will it take him to travel 600 miles, when the day is 16 hours long?

$$\begin{array}{rclclcl} & \text{miles} & & \text{days} & & \text{miles.} \\ \text{As } + & 300 & : & 10 & :: & 600 \\ & & & 12 \text{ hrs.} & : & - & :: & +16 \text{ hours.} \end{array}$$

NOTE.—In the second stating, the day being 16 hours long, more requires less; consequently, this stating is inverse, and the 16 must be marked for a division.

$$\text{Then } \frac{10 \times 600 \times 12}{300 \times 16} = \frac{72000}{4800} = 15 \text{ days. } \text{Ans.}$$

3. If 248 men, in 5 days, each 11 hours long, can dig a trench 230 yards long, 3 wide, and 2 deep; in how many days, each 9 hours long, will 24 men dig a trench 420 yards long, 5 wide, and three deep?

$$\begin{array}{rclclcl} \text{As } 248 \text{ men} & : & 5 \text{ days} & :: & + & 24 \text{ men.} \\ & & 11 \text{ hours} & : & \text{—} & :: & + & 9 \text{ hours.} \\ + & 230 \text{ yards} & : & \text{—} & :: & 420 \text{ yards.} \\ + & 3 \text{ yds. wide} & : & \text{—} & :: & 5 \text{ yds wide.} \\ + & 2 \text{ yds. deep} & : & \text{—} & :: & 3 \text{ yds deep.} \end{array}$$

NOTE.—Here the first and second statings are inverse, and the other three direct, and the divisors marked accordingly.

$$\text{Here } \frac{248 \times 5 \times 11 \times 420 \times 5 \times 3}{24 \times 9 \times 230 \times 3 \times 2} = \frac{85932000}{298080} = 288 \text{ days } 2\frac{1}{3}$$

Or by Rule 2.

$$\begin{array}{l} \text{Here } 5 \times 11 = 55 \text{ hours.} \\ 230 \times 3 \times 2 = 1380 \text{ cubic yards.} \\ 420 \times 5 \times 3 = 6300 \text{ do.} \end{array}$$

$$\begin{array}{rclclcl} & \text{men.} & & \text{hours.} & & \text{yards.} \\ \text{As } 248 & : & 55 & :: & 1380 \\ & 24 & : & \text{—} & :: & 6300 \end{array}$$

$$\text{Then } \frac{248 \times 55 \times 6300}{1380 \times 24} = \frac{85932000}{33120} = 2594\frac{1}{3} \text{ h.} = 288 \text{ d } 2\frac{1}{3}. \text{Ans.}$$

PRACTICAL EXAMPLES.

1. If 2 bushels of oats will serve 6 horses 4 days; how many bushels will serve 42 horses 12 days? *Ans. 42 bush.*

2. If 4 men spend £3 in 5 days; how much will 7 men spend in 12 days? *Ans.* £12 12s.

3. If 12 men spend £12 12s. in 12 days; how much will 4 men spend in 5 days? *Ans.* £1 15s.

4. If 10 men can mow 60 acres of grass in 3 days; how many acres can 7 men mow in 8 days? *Ans.* 112 acres.

5. If 7 men can mow 112 acres of corn in 8 days; how many acres can 10 men mow in 3 days? *Ans.* 60 acres.

6. If £100 in 12 months gain £5; what will £75 gain in 9 months? *Ans.* £2 16s. 3d.

7. If £75 gain £2 16s. 3d. in 9 months; how much will £100 gain in 12 months? *Ans.* £5.

8. If a carrier receives £6 13s. 4d. for the carriage of 11 cwt. 2 qrs. 160 miles; what will be the expense of 15 cwt. 1 qr. 14 lb. for 64 miles? *Ans.* £3 11s. 3½d.

9. If a regiment of 1878 soldiers consume 702 quarters of wheat in 336 days; how many quarters will 11268 soldiers consume in 112 days? *Ans.* 1404 qrs.

10. If 20 measures of wine, each 3s. 6d., serve 14 men 6 days; how many measures, 2s. 6d. each, will serve 60 men 4 days? *Ans.* 80.

11. If £360 amount to £369 2s. in seven months; what is the rate per cent, per annum? *Ans.* £4 6s. 8d.

12. If a beam of wood 15 feet long, 15 inches deep, and 12 inches broad, weigh 600 lbs.; how much will a beam of the same kind of wood weigh, that is 20 feet long, 21 inches deep, and 15 inches broad? *Ans.* 1400 lbs.

13. If a garrison of 3600 men consume 540 qrs. of wheat in 35 days, at 24 oz. for each man per day; how many men may live 50 days on 810 quarters, at the rate of 1 lb. for each man per day? *Ans.* 5670.

14. Paid £3 for the carriage of 200 cubic feet of wood 50 miles, each foot weighing 48 oz.; how much will the carriage of 70 cubic feet of stone, that weighs 10 stone per foot, cost for 30 miles? *Ans.* £2 2s.

15. If 4 compositors, in 16 days of 12 hours long, can compose 14 sheets, of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line; how many days, each 10 hours, will 9 compositors require to compose a volume of the same type, consisting of 30 sheets, 16 pages in a sheet, 48 lines in a page, and 45 letters in a line? *Ans.* 14 days 9 hrs. 36½.

PRACTICE.

PRACTICE is a short method of working the Rule of Three; when the first term is a unit or one.

A TABLE OF ALIQUOT PARTS.

6d. is $\frac{1}{2}$ of a shilling.	10s. 0d. is $\frac{1}{2}$ of a pound.
4d. is $\frac{1}{3}$ _____	6s. 8d. is $\frac{1}{3}$ _____
3d. is $\frac{1}{4}$ _____	5s. 0d. is $\frac{1}{4}$ _____
2d. is $\frac{1}{6}$ _____	4s. 0d. is $\frac{1}{5}$ _____
1½d. is $\frac{1}{8}$ _____	3s. 4d. is $\frac{1}{6}$ _____
1d. is $\frac{1}{12}$ _____	2s. 6d. is $\frac{1}{5}$ _____
½d. is $\frac{1}{24}$ of a penny.	2s. 0d. is $\frac{1}{10}$ _____
¼d. is $\frac{1}{48}$ _____	1s. 8d. is $\frac{1}{15}$ _____
	1s. 0d. is $\frac{1}{20}$ _____
10 cwt. is $\frac{1}{2}$ of a ton	2 qrs. is $\frac{1}{4}$ of a cwt.
5 cwt. is $\frac{1}{4}$ _____	1 qr. is $\frac{1}{8}$ _____
4 cwt. is $\frac{1}{5}$ _____	16 lb. is $\frac{1}{8}$ _____
2 cwt. is $\frac{1}{10}$ _____	14 lb. is $\frac{1}{7}$ _____
1 cwt. is $\frac{1}{20}$ _____	8 lb. is $\frac{1}{14}$ _____
	7 lb. is $\frac{1}{16}$ _____
14 lb. is $\frac{1}{4}$ of a quarter.	8 oz. is $\frac{1}{4}$ of a lb.
7 lb. is $\frac{1}{8}$ _____	4 oz. is $\frac{1}{8}$ _____
4 lb. is $\frac{1}{16}$ _____	2 oz. is $\frac{1}{16}$ _____
3½ lb. is $\frac{1}{24}$ _____	1 oz. is $\frac{1}{32}$ _____

CASE I.—When the given price is less than a penny.

RULE—If the given price be an aliquot part of a penny, as in the first example, divide the given number by that aliquot part, and the quotient will be the answer in pence, which divide by 12 and 20. If the given price be not an aliquot part, proceed as in example 2nd. Or, multiply the given number by the price, and divide the product by 4, 12, and 20.

EXAMPLES.

1. What is the value of 3652 yards, at ½d. per yard?

$$\begin{array}{r}
 \frac{1}{2}d. \text{ is } \frac{1}{2} \quad 3652 \\
 \hline
 12)1826 \\
 \hline
 2,0)15,2 \quad 2 \\
 \hline
 \pounds 7 \quad 12 \quad 2 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 \text{or} \quad 3652 \\
 \hline
 2 \\
 \hline
 4)7304 \\
 \hline
 12)1826 \\
 \hline
 20)152 \quad 2 \\
 \hline
 \pounds 7 \quad 12 \quad 2
 \end{array}$$

2. What is the value of 4573 yards at $\frac{3}{4}$ d. per yard?

$$\frac{1}{2} \text{ is } \frac{1}{2})4573$$

$$\begin{array}{r} 4573 \\ 3 \end{array}$$

$$\frac{1}{4} \text{ is } \frac{1}{4})2286 \frac{1}{2}$$

$$4)13719$$

$$12)3429 \frac{1}{2}$$

$$\text{Or } 12)3429 \frac{1}{2}$$

$$2,0)285 \ 9\frac{1}{2}$$

$$2,0)285 \ 9\frac{1}{2}$$

$$\pounds 14 \ 5 \ 9\frac{3}{4} \text{ Ans.}$$

$$\pounds 14 \ 5 \ 9\frac{1}{2} \text{ Ans.}$$

$$3. \quad 5702 \text{ at } \frac{1}{4}\text{d.}$$

$$\pounds \ s. \ d. \\ \text{Ans. } 5 \ 18 \ 9\frac{1}{2}$$

$$4. \quad 7696 \text{ at } \frac{1}{2}\text{d.}$$

$$16 \ 0 \ 8$$

$$5. \quad 6546 \text{ at } \frac{3}{4}\text{d.}$$

$$20 \ 9 \ 1\frac{1}{2}$$

CASE II. *When the price is a penny, or pence and farthings, and does not exceed one shilling.*

RULE.—If the given price be an aliquot part of a shilling, as in example 1st, divide the given number by the aliquot part, and the quotient will be the answer in shillings, which divide by 20.

If the given price be not any aliquot part of a shilling, divide the given price into parts, taking the first an aliquot or even part of a shilling, and the remainder into aliquot parts of this and so on, as in example 2nd, and the sum of these quotients will be the answer in shillings, which divide by 20.

Or, multiply the given number by the price, and the product will be the answer in pence, which divide by 12 and 20.

EXAMPLES.

1. What is the value of 365 yards, at 2d. per yard?

$$2\text{d. is } \frac{1}{5})365$$

$$\text{Or } \begin{array}{r} 365 \\ 2 \end{array}$$

$$2,0)60 \ 10$$

$$12)730$$

$$\pounds 3 \ 0 \ 10 \text{ Ans.}$$

$$20)60 \ 10$$

$$\pounds 3 \ 0 \ 10$$

2. What is the value of 468 yards, at $5\frac{1}{4}$ d. per yard?

$$\begin{array}{r} 4\text{d. is } \frac{1}{4})468 \\ \hline \end{array}$$

$$\begin{array}{r} 1\text{d. is } \frac{1}{4})156 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{4}\text{d. is } \frac{1}{4} \quad 39 \\ \hline 9 \quad 9 \end{array}$$

$$\begin{array}{r} 2,0)204 \quad 9 \\ \hline \end{array}$$

$$\text{£}10 \quad 4 \quad 9 \text{ Ans.}$$

$$\text{Or, } \begin{array}{r} 468 \\ \hline \end{array}$$

$$\begin{array}{r} 5\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 2340 \\ \hline \end{array}$$

$$\begin{array}{r} 117 \\ \hline \end{array}$$

$$\begin{array}{r} 12)2457 \\ \hline \end{array}$$

$$\begin{array}{r} 2,0)204 \quad 9 \\ \hline \end{array}$$

$$\text{£}10 \quad 4 \quad 9 \text{ Ans.}$$

3. What is the value of 468 yards, at $5\frac{1}{2}$ d. per yard?

$$\begin{array}{r} 4\text{d. is } \frac{1}{2})468 \\ \hline \end{array}$$

$$\begin{array}{r} 1\frac{1}{2} \text{ is } \frac{1}{2} 156 \\ \hline \end{array}$$

$$\begin{array}{r} 58 \quad 6 \\ \hline \end{array} \left\{ \begin{array}{l} \text{this is } \frac{1}{2} \text{ of the given number,} \\ 1\frac{1}{2}\text{d, being } \frac{1}{2} \text{ of a shilling.} \end{array} \right.$$

$$\begin{array}{r} 2,0)214 \quad 6 \\ \hline \end{array}$$

$$\text{£}10 \quad 14 \quad 6$$

$$4. \quad 7546 \text{ at } 1\text{d.}$$

$$\text{Ans. £}31 \quad 8 \quad 10$$

$$13. \quad 3531 \text{ at } 3\frac{1}{4}\text{d.}$$

$$\text{Ans. £}47 \quad 16 \quad 3\frac{1}{4}$$

$$22. \quad 3121 \text{ at } 5\frac{1}{2}\text{d.}$$

$$\text{Ans. £}71 \quad 10 \quad 5\frac{1}{2}$$

$$5. \quad 7546 \text{ at } 1\frac{1}{2}\text{d.}$$

$$\text{Ans. £}39 \quad 6 \quad 0\frac{1}{2}$$

$$14. \quad 4294 \text{ at } 3\frac{1}{2}\text{d.}$$

$$\text{Ans. £}62 \quad 12 \quad 5$$

$$23. \quad 7520 \text{ at } 5\frac{3}{4}\text{d.}$$

$$\text{Ans. £}180 \quad 3 \quad 4$$

$$6. \quad 54326 \text{ at } 1\frac{1}{2}\text{d.}$$

$$\text{Ans. £}339 \quad 10 \quad 9$$

$$15. \quad 3500 \text{ at } 3\frac{3}{4}\text{d.}$$

$$\text{Ans. £}54 \quad 13 \quad 9$$

$$24. \quad 4271 \text{ at } 6\text{d.}$$

$$\text{Ans. £}106 \quad 15 \quad 6$$

$$7. \quad 6255 \text{ at } 1\frac{3}{4}\text{d.}$$

$$\text{Ans. £}45 \quad 12 \quad 2\frac{1}{4}$$

$$16. \quad 3258 \text{ at } 4\text{d.}$$

$$\text{Ans. £}54 \quad 6 \quad 0$$

$$25. \quad 3252 \text{ at } 6\frac{1}{2}\text{d.}$$

$$\text{Ans. £}88 \quad 1 \quad 6$$

$$8. \quad 2350 \text{ at } 2\text{d.}$$

$$\text{Ans. £}19 \quad 11 \quad 8$$

$$17. \quad 2057 \text{ at } 4\frac{1}{4}\text{d.}$$

$$\text{Ans. £}36 \quad 8 \quad 6\frac{1}{4}$$

$$26. \quad 2710 \text{ at } 6\frac{3}{4}\text{d.}$$

$$\text{Ans. £}76 \quad 4 \quad 4\frac{1}{2}$$

$$9. \quad 7211 \text{ at } 2\frac{1}{4}\text{d.}$$

$$\text{Ans. £}67 \quad 12 \quad 0\frac{3}{4}$$

$$18. \quad 1028 \text{ at } 4\frac{1}{2}\text{d.}$$

$$\text{Ans. £}19 \quad 5 \quad 6$$

$$27. \quad 2702 \text{ at } 7\frac{1}{2}\text{d.}$$

$$\text{Ans. £}84 \quad 8 \quad 9$$

$$10. \quad 2711 \text{ at } 2\frac{1}{2}\text{d.}$$

$$\text{Ans. £}28 \quad 4 \quad 9\frac{1}{2}$$

$$19. \quad 2108 \text{ at } 4\frac{3}{4}\text{d.}$$

$$\text{Ans. £}41 \quad 14 \quad 5$$

$$28. \quad 2713 \text{ at } 8\text{d.}$$

$$\text{Ans. £}90 \quad 8 \quad 8$$

$$11. \quad 3249 \text{ at } 2\frac{3}{4}\text{d.}$$

$$\text{Ans. £}37 \quad 4 \quad 6\frac{1}{4}$$

$$20. \quad 3211 \text{ at } 5\text{d.}$$

$$\text{Ans. £}66 \quad 17 \quad 11$$

$$29. \quad 9873 \text{ at } 8\frac{1}{2}\text{d.}$$

$$\text{Ans. £}359 \quad 19 \quad 0\frac{1}{2}$$

$$12. \quad 2714 \text{ at } 3\text{d.}$$

$$\text{Ans. £}33 \quad 18 \quad 6$$

$$21. \quad 2714 \text{ at } 5\frac{1}{4}\text{d.}$$

$$\text{Ans. £}59 \quad 7 \quad 4\frac{1}{2}$$

$$30. \quad 7925 \text{ at } 9\frac{1}{2}\text{d.}$$

$$\text{Ans. £}313 \quad 13 \quad 11\frac{1}{2}$$

31. 5725 at $10\frac{1}{4}$ d.	33. 7255 at $11\frac{1}{4}$ d.	35. 7973 at $11\frac{3}{4}$ d.
Ans. £244 10 1 $\frac{1}{4}$	Ans. £340 1 6 $\frac{3}{4}$	Ans. £390 6 10 $\frac{1}{4}$
32. 7290 at $10\frac{3}{4}$ d.	34. 3755 at $11\frac{1}{4}$	
Ans. £326 10 7 $\frac{1}{2}$	Ans. £179 18 6 $\frac{1}{4}$	

CASE III. When the price is more than one shilling, but less than two.

RULE.—Let the given number stand for shillings, and work for the pence and farthings as in the second case.

EXAMPLES.

1. What is the value of 961 yards, at 1s. $0\frac{1}{4}$ d. per yard.

$$\begin{array}{r}
 \frac{1}{4}\text{d. is } \frac{1}{48} \quad 961 \\
 \quad \quad \quad 20 \quad 0\frac{1}{4} \\
 \hline
 2,0)981 \quad 0\frac{1}{4} \\
 \hline
 \text{£49 } 1 \quad 0\frac{1}{4} \text{ Ans.}
 \end{array}$$

2. What is the value of 2436 yards at 1s. $0\frac{1}{2}$ d. per yard?

$$\begin{array}{r}
 \frac{1}{2} \text{ is } \frac{1}{24} \quad 2436 \\
 \quad \quad \quad 101 \quad 6 \\
 \hline
 2,0)2537 \quad 6 \\
 \hline
 \text{£126 } 17 \quad 6 \text{ Ans.}
 \end{array}$$

3. What is the value of 468 yards, at 1s. $7\frac{1}{4}$ d. per yard?

$$\begin{array}{r}
 6\text{d. is } \frac{1}{2} \quad 468 \\
 1\frac{1}{4}\text{d. is } \frac{1}{4} \quad 234 \\
 \quad \quad \quad 58 \quad 6 \\
 \hline
 2,0)760 \quad 6 \\
 \hline
 \text{£38 } 0 \quad 6 \text{ Ans.}
 \end{array}$$

4. 478 at 1s. $0\frac{3}{4}$ d.	8. 575 at 1s. $1\frac{1}{4}$ d.	12. 3625 at 1s. $2\frac{3}{4}$ d.
Ans. £25 7 10 $\frac{3}{4}$	Ans. £32 18 10 $\frac{1}{4}$	Ans. £222 15 8 $\frac{3}{4}$
5. 575 at 1s. 1d.	9. 990 at 1s. 2d.	13. 7590 at 1s. 3d.
Ans. £31 2 11	Ans. £57 15	Ans. £474 7 6
6. 676 at 1s. $1\frac{1}{4}$ d.	10. 3290 at 1s. $2\frac{1}{4}$ d.	14. 6326 at 1s. $3\frac{1}{4}$ d.
Ans. £37 6 5	Ans. £195 6 10 $\frac{1}{4}$	Ans. £401 19 3 $\frac{1}{4}$
7. 707 at 1s. $1\frac{1}{2}$ d.	11. 4627 at 1s. $2\frac{1}{2}$ d.	15. 5272 at 1s. $3\frac{1}{2}$ d.
Ans. £39 15 4 $\frac{1}{2}$	Ans. £279 10 11 $\frac{1}{2}$	Ans. £340 9 8

16. 1627 at 1s. 3½d. <i>Ans.</i> £106 15 5½	26. 7102 at 1s. 6½d. <i>Ans.</i> £540 0 11½	36. 1005 at 1s. 8¾d. <i>Ans.</i> £86 17 9¾
17. 2914 at 1s. 4d. <i>Ans.</i> £194 5 4	27. 1627 at 1s. 6½d. <i>Ans.</i> £125 8 3½	37. 2105 at 1s. 9d. <i>Ans.</i> £184 3 9
18. 1635 at 1s. 4½d. <i>Ans.</i> £110 14 0¾	28. 7924 at 1s. 6¾d. <i>Ans.</i> £619 1 3	38. 2570 at 1s. 9¼d. <i>Ans.</i> £227 11 0½
19. 7060 at 1s. 4½d. <i>Ans.</i> £485 7 6d.	29. 9272 at 1s. 7d. <i>Ans.</i> £734 0 8	39. 1052 at 1s. 9½d. <i>Ans.</i> £94 4 10
20. 1375 at 1s. 4½d. <i>Ans.</i> £95 19 3½	30. 3605 at 1s. 7¼d. <i>Ans.</i> £289 3 0½	40. 3753 at 1s. 9½d. <i>Ans.</i> £340 2 3½
21. 3726 at 1s. 5d. <i>Ans.</i> £263 18 6	31. 1155 at 1s. 7½d. <i>Ans.</i> £93 16 10½	41. 1072 at 1s. 10d. <i>Ans.</i> £98 5 4
22. 3625 at 1s. 5½d. <i>Ans.</i> £260 10 11½	32. 1252 at 1s. 7¾d. <i>Ans.</i> £103 0 7	42. 2600 at 1s. 10¼d. <i>Ans.</i> £241 0 10
23. 2596 at 1s. 5½d. <i>Ans.</i> £189 5 10	33. 1753 at 1s. 8d. <i>Ans.</i> £146 1 8	43. 2116 at 1s. 10½d. <i>Ans.</i> £198 7 6
24. 3605 at 1s. 5¾d. <i>Ans.</i> £266 12 4½	34. 2906 at 1s. 8¼d. <i>Ans.</i> £245 3 10½	44. 2014 at 1s. 10¾d. <i>Ans.</i> £190 18 2½
25. 3762 at 1s. 6d. <i>Ans.</i> £282 3	35. 3552 at 1s. 8½d. <i>Ans.</i> £303 8	45. 7854 at 1s. 11¾d. <i>Ans.</i> £777 4 4½

CASE IV. *When the given price is any number of shillings.*

RULE.—Multiply the given number by the price, and the product will be the answer in shillings, which divide by 20.

If the given price be any even number of shillings, multiply the given number by half the price, double the product of the first figure for shillings, and the remainder of the product will be pounds.

If the given price be any aliquot part of a pound, divide by that aliquot part, and the quotient will be the answer.

EXAMPLES.

1. What is the value of 366 yards, at 4s. per yard.

366 4 — 2,0)1464 — £73 4 Ans.	Or, 366 2 — £73 4 Ans.	Or, 4s. is ¼)366 — £73 4 Ans.
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2. What is the value of 492 yards, at 8s. per yard ?

$\begin{array}{r} 492 \\ 8 \overline{) 20)3936} \end{array}$	Or, $\begin{array}{r} 492 \\ 4 \overline{) 196\ 16} \end{array}$	Or, 4s. is $\frac{1}{2}$ 492 $\begin{array}{r} 4s. \text{ is } \frac{1}{2} \ 98 \ 8 \\ \phantom{4s. \text{ is } \frac{1}{2}} \ 98 \ 8 \end{array}$
£196 16 Ans.	£196 16 Ans.	£196 16 Ans.

3. 1375 at 2s. Ans. £137 10	9. 786 at 8s. Ans. £314 8	15. 5272 at 14s. Ans. £3690 8
4. 1635 at 3s. Ans. £245 5	10. 1607 at 9s. Ans. £723 3	16. 2151 at 15s. Ans. £1613 5
5. 1627 at 4s. Ans. £325 8	11. 4561 at 10s. Ans. £2280 10	17. 1074 at 16s. Ans. £859 4
6. 1645 at 5s. Ans. £411 5	12. 1355 at 11s. Ans. £745 5	18. 9143 at 17s. Ans. £2671 11
7. 1355 at 6s. Ans. £406 10	13. 2102 at 12s. Ans. £1261 4	19. 1622 at 18s. Ans. £1459 16
8. 2714 at 7s. Ans. £949 18	14. 3178 at 13s. Ans. £2065 14	20. 2151 at 19s. Ans. £2043 9

CASE V.—When the price is shillings and pence.

RULE.—Multiply the given number by the shillings, and take aliquot parts with the pence, add them together, and divide the sum by 20.

Or: Divide the price into aliquot parts of a pound, or take a part of the price which may be an aliquot part of a pound, and divide the remainder into aliquot parts of this or of each other, and the sum of the quotients, belonging to each, will be the answer required.

EXAMPLES.

1. What is the value of 365 yards, at 3s. 9d. per yard ?

$\begin{array}{r} 6d. \text{ is } \frac{1}{2} \ 365 \\ \phantom{6d. \text{ is } \frac{1}{2}} \ 3 \overline{) 1095} \\ 3 \text{ is } \frac{1}{3} \ 182 \ 6 \\ \phantom{3 \text{ is } \frac{1}{3}} \ 91 \ 3 \overline{) 2,0)136,8 \ 9} \\ \phantom{3 \text{ is } \frac{1}{3}} \end{array}$	$\begin{array}{r} \text{Or } 2s. \ 6d. \text{ is } \frac{1}{8} (265 \\ \phantom{\text{Or } 2s. \ 6d. \text{ is } \frac{1}{8}} \phantom{1s. \ 3d. \text{ is } \frac{1}{4}} \\ 1s. \ 3d. \text{ is } \frac{1}{4} \ 45 \ 12 \ 6 \\ \phantom{1s. \ 3d. \text{ is } \frac{1}{4}} \\ \phantom{1s. \ 3d. \text{ is } \frac{1}{4}} \end{array}$
£68 8 9	£68 8 9 Ans.

2. What is the value of 456 yards, at 3s. 11½d. per yard?

$$\begin{array}{r} 6d. \text{ is } \frac{1}{2} \quad 456 \\ 4d. \text{ is } \frac{1}{3} \quad 3 \end{array}$$

$$\begin{array}{r} 1368 \\ 1\frac{1}{2}d. \text{ is } \frac{1}{4} \quad 228 \\ 152 \\ 57 \end{array}$$

$$\begin{array}{r} 2,0)180,5 \end{array}$$

£90 5 Ans.

$$\text{Or, } 3s. 4d. \text{ is } \frac{1}{2})456$$

$$\begin{array}{r} 5d. \text{ is } \frac{1}{4} \quad 76 \\ 2\frac{1}{2}d. \text{ is } \frac{1}{2} \quad 9 \quad 10 \\ 4 \quad 15 \end{array}$$

£90 5 Ans.

3. 374 at 1s. 8d.	8. 3714 at 9s. 4½d.	13. 3211 at 15s. 7¾d.
Ans. £31 3 4	Ans. £1740 18 9	Ans. £2511 18 9½
4. 5681 at 2s. 6d.	9. 1286 at 13s. 7½d.	14. 7153 at 17s. 6¾d.
Ans. £710 2 6	Ans. £876 1 9	Ans. £6281 4 6¾d.
5. 7856 at 3s. 4d.	10. 2417 at 14s. 8¼d.	15. 4568 at 19s. 11½d.
Ans. £1309 6 8	Ans. £1774 19 8¼	Ans. £4563 4 10
6. 325 at 6s. 8d.	11. 1604 at 16s. 10½d.	
Ans. £108 6 8	Ans. £1353 7 6	
7. 2548 at 7s. 3½d.	12. 1355 at 19s. 2½d.	
Ans. £928 19 2	Ans. £1301 7 3½	

CASE VI.---When the price is pounds and shillings, or pounds, shillings and pence.

RULE.---Multiply the given numbers by the pounds, and work with the rest by the former rules; add these quantities together for the answer.

EXAMPLES.

1. What is the value of 372 cwt. at £2 7s. 6d. per cwt.?

$$\begin{array}{r} 5s. \text{ is } \frac{1}{2} \quad 372 \\ 2 \\ \hline 744 \\ 2s. 6d. \text{ is } \frac{1}{4} \quad 93 \\ 46 \quad 10 \end{array}$$

£883 10 Ans.

2. What is the value of 564 cwt. at £3 4s. 4½d. per cwt ?

$$\begin{array}{r} 4s. \text{ is } \frac{1}{2} \quad 564 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 1692 \\ 4d. \text{ is } \frac{1}{4} \quad 112 \quad 16 \\ \frac{1}{2}d. \text{ to } \frac{1}{2} \quad 9 \quad 8 \\ \hline 1 \quad 3 \quad 6 \end{array}$$

£1815 7 6 Ans.

3. 2702 at £2. 3s. 4d.	5. 719 at £2. 7s. 4½d.	7. 2150 at £8 18s. 0½d
Ans. £5854 6 8	Ans. £1703 2 7½	Ans. £19141 14 4½
4. 839 at £3. 15s. 2½d.	6. 3209 at £1 18s 6½d	8. 1355 at £2 3s. 7½d
Ans. £3154 2 3½	Ans. £6187 7 0½	Ans. £2955 11 10½

CASE VII.---When the given number contains a fractional part.

RULE.---Find the value of the given number, without the fractional part, as in the foregoing rules, to which add the value of the fractional part which may be found as in compound division.

EXAMPLES.

1. What is the value of 959¾ yards at 6s. 8d. per yard ?

$$6s. 8d. \text{ is } \frac{1}{2} \quad 959\frac{3}{4}$$

$$\begin{array}{r} 319 \quad 13 \quad 4 \\ 0 \quad 10 \text{ is } \frac{1}{2} \text{ of } 6s. 8d. \\ 1 \quad 8 \text{ is } \frac{1}{2} \text{ of } 6s. 8d. \end{array}$$

£319 15 10 Ans.

2. What is the value of 456¾ yards 4s. 10d. per yard ?

$$\begin{array}{r} 6d. \text{ is } \frac{1}{4} \quad 456\frac{3}{4} \\ \hline 4 \end{array}$$

$$\begin{array}{r} 4d. \text{ is } \frac{1}{2} \quad 1824 \\ 228 \\ 152 \end{array}$$

$$\begin{array}{r} \frac{1}{2} \text{ of } 4s. 10d. = 0 \quad 7\frac{1}{2} \times 6 \\ \frac{1}{4} \text{ of } 4s. 10d. = 3 \quad 7\frac{1}{2} \end{array}$$

$$2,0)2208 \quad 2\frac{1}{2}$$

£110 8 2½ Ans.

3. $677\frac{1}{2}$ at 6s. 8d.	6. $834\frac{1}{2}$ at 4s. 7d.	9. $314\frac{1}{2}$ at £3 15s. 2½d.
<i>Ans.</i> £225 16 8	<i>Ans.</i> £191 6 6½d.	<i>Ans.</i> £1182 15 10½
4. $339\frac{1}{2}$ at 2s. 6d.	7. $901\frac{1}{2}$ at £7. 4s.	10. $357\frac{1}{2}$ at £4 6 8
<i>Ans.</i> £42 8 5½	<i>Ans.</i> £6493 10	<i>Ans.</i> £1547 19 3
5. $1021\frac{1}{2}$ at 1s. 8d.	8. $604\frac{1}{2}$ at 14s. 8½d.	11. $346\frac{1}{2}$ at £33 6s. 8d.
<i>Ans.</i> £85 2 4½	<i>Ans.</i> £443 14 11	<i>Ans.</i> £11561 2 2½ ½

CASE VIII.—When the given quantity is of several denominations.

RULE.—Multiply the price by the highest denomination, and take aliquot parts for the lower denominations, which add to the former product, and the sum will be the answer.

In compositions and bankruptcies, divide the given sum or debt by the aliquot parts of a pound, contained in the composition or sum paid in the pound, and the sum of the quotients will be the amount of the composition required.

EXAMPLES.

1. What is the value of 9 cwt. 3 qrs. 15 lb. of sugar, at £2 18s. 8d. per cwt.?

<i>Qrs.</i>		£.	s.	d.
2	½	2	18	8
				9
1	½	26	8	0
8 lb.	¾	1	9	4
7	½		14	8
			4	2½
			3	8
		28	19	10½ <i>Ans.</i>

2. A person who owes me £284 12s. 6d. fails; how much shall I receive for the same, at 5s. 4d. in the pound?

<i>s.</i>		£	s.	d.
4	½	284	12	6
1s. 4d.	¾	56	18	6
		18	19	6
		75	18	0 <i>Ans.</i>

3. What is the value of 24 cwt. 2 qrs. 14 lb. of sugar, at £3 17s. 6d. per cwt?

Ans. £95 8 5½

4. What is the value of 30 cwt. 2 qrs. 20 lb. at £1 17s. 4d. per cwt?

Ans. £57 5s. 4d.

5. What is the value of 27 cwt. 3 qrs, 7 lb. of soap, at £3 5s. 4d. per cwt? *Ans.* £90 17s. 1d.

6. What is the value of 9 ton, 9 cwt. 3 qr. 14 lb. of hay, at £5 per ton? *Ans.* £47 9s. 4½d.

7. What is the value of 3 qrs, 21 lb, at £11 12s. 8d. per cwt? *Ans.* £10 18s. 1½d.

8. What is the value of 37 oz, 1 dwt, 6 gr of silver, at 5s. per oz? *Ans.* £9 5s 3½d

9. What is the value of a silver pint, weighing 7 oz, 11 dwt, at 8s. 11d. per oz? *Ans.* £3 7s. 3½d

10. What is the value of 257 yards, 1 qr, 2 nails, at 17s. 8d per yard? *Ans.* £227 6s. 11½d

11. What is the value of 250 E Ells, 2 qr, 2 n., at 15s. 10½d per Ell? *Ans.* £199 1s 10¾d

12. What is the value of 51 French Ells, 5 qrs, 1 n., at £1 6s. 6d per Ell? *Ans.* £68 14s 8½d

13. What is the half year's rent on 103 a, 2 r, 30 p, at £2 5s per acre per annum? *Ans.* 116 12s 11½d

14. What is the value of 83 gal, 3 quarts, 1 p. of brandy, at £1 9s. 6d per gallon? *Ans.* £123 14s 3¾d

15. A bankrupt owes me £136 12s. 6d.; how much shall I receive for the same, at 7s. 9¾d. in the pound? *Ans.* £53 7s. 4½d

TARE AND TRET.

TARE AND TRET are rules for deducting customary allowances in business, when goods are sold by weight.

1. Gross weight is the weight of the goods, and the box, barrel, or bag which contains them.

2. Tare is an allowance made to the buyer for the weight of the box, barrel, bag, &c. which contains the goods; and is generally at so much per box, barrel, &c.; but in some cases, at so much per cwt.; or at so much per cent, that is, so many lb. in every 100 lb.

3. Suttle is the weight of the goods, when the tare is taken from the gross weight, and is called Tare Suttle.

4. Tret is an allowance of 4 lb. in every 104, or $\frac{1}{26}$ part of the whole, for waste, &c.

5. Cloff is an allowance made, after tare and tret are deducted, of 2 lb. upon every 3 cwt., that the weight may hold good, when the article is sold by retail.

6. Neat weight is what remains after all allowances are made.

CASE I.—*When the tare is so much for the whole.*

RULE.—Subtract the given tare from the gross weight, and the remainder will be the neat weight.

EXAMPLES.

1. What is the neat weight of a cask of raisins, weighing 3 cwt. 2 qrs. 15 lb. tare being 1 qr. 18 lb ?

$$\begin{array}{r} \text{cwt. qr. lb.} \\ 3 \quad 2 \quad 15 \\ \quad \quad 1 \quad 18 \\ \hline \end{array}$$

3 0 25 neat weight.

2. What is the neat weight of a barrel of figs, weighing 4 cwt. 3 qrs. 2 lb., tare being 1 qr. 17 lb ?

Ans. 4 cwt. 1 qr. 13 lb.

3. What is the neat weight of 3 hogsheads of tobacco, weighing 57 cwt. 3 qrs. 16 lb, gross, tare being 4 cwt. 3 qrs. 18 lb ?

Ans. 52 cwt. 3 qr. 26 lb.

4. A cart load of hay was found to weigh 32 cwt. 3 qrs. 14 lb. including the cart, weighing 15 cwt. 1 qr. 25 lb. ; what did the hay alone weigh ?

Ans. 17 cwt. 1 qr. 17 lb.

CASE II.—*When the tare is at so much per cask, box, bag, &c.*

RULE.—Find the neat weight of one cask, box, bag, &c. by the first case, which multiply by the number of casks, &c. and the product will be the neat weight. The value may be found by Practice.

EXAMPLES.

1. What is the neat weight of 8 boxes of figs, each weighing 1 cwt. 3 qr. 14 lb. gross, tare or weight of the box alone being 21 lb ?

$$\begin{array}{r} \text{cwt. qr. lb.} \\ 1 \quad 3 \quad 14 \\ \quad \quad 21 \text{ tare.} \\ \hline \end{array}$$

1 2 21 neat weight of 1 box.
8

13 2 0 neat weight of 8 boxes.

2. Bought 7 casks of sugar, each weighing 5 cwt. 2 qr. 5 lb. gross, tare 1 qr. 18 lb. per cask ; what is the neat weight, and its value at 56 shillings per cwt.

Ans. { 35 cwt, 3 qr. 21 lb.
£100 12s. 6d.

3. Bought 12 barrels of figs, each 3 cwt. 1 qr. 7 lb. gross, tare 1 qr. 14 lb. per barrel; what is the neat weight, and the cost of the whole, at £3 5s. 4d. per cwt?

Ans. { 35 cwt. 1 qr.
£115 3s.

CASE III.—When the tare is at so much per cwt., or per cent.

RULE.—If the tare be any aliquot part of a cwt., divide the gross weight by this aliquot part, and the quotient will be the tare, which subtract from the gross weight, and the remainder will be the neat weight required.

When the tare is at so much per cent., that is, so many lb. for every 100 lb., it may be found by the Rule of Three, or by taking aliquot parts.

EXAMPLES.

1. What is the neat weight of $10\frac{1}{2}$ boxes of soap, each weighing 1 cwt. 1 qr. 23 lb. gross, tare 13 lb. per cwt.?

cwt. qrs. lbs.

1 1 23
10½

14 2 6
2 25 8 oz.

8 lb. $\frac{1}{4}$ is 15 1 3 8 gross weight.

4 lb. is $\frac{1}{2}$ 1 0 10 4

1 lb. is $\frac{1}{4}$ 2 5 2

15 4 8 drs.

1 3 2 10 8 Tare

13 2 0 13 8 *Ans.*

2. Bought 36 casks of treacle, each 3 cwt. 2 qrs. 14 lb. gross, tare per cwt. 8 lb.; what is the neat weight of the whole, and what will the same cost, at £1 2s. 6d. per cwt.?

Ans. { 121 cwt. 0 qr. 20 lb. neat weight.
£136 6s. 6d. cost.

3. Bought 25 cwt. 3 qr. 12 lb. gross tare 14 lb. per cwt.; required its neat weight and value, at £2 16s. per cwt.;

Ans. { 22 cwt. 2 qrs. 14 lb.
£63 7s. value.

4. Required the value of 3600 lb. of wool, at 1s. 8d. per lb., tare being allowed at 4 lb per cent. *Ans.* £288.

CASE IV.—*To find the neat weight when tret is allowed with tare.*

RULE.—Find the neat weight as above, and call it tareuttle; which divide by 26, and subtract the quotient from it, and the remainder will be the neat weight.

EXAMPLES.

1. Bought 4 hhds. of sugar, each weighing 9 cwt., tare of the whole 2 cwt. 2 qr. 8 lb., tret 4 lb. for 104 lb.; how much neat weight?

cwt.	qr.	lb.
9	0	0
		4

36	0	0	whole gross weight.
2	2	8	tare.

26)33	1	20	tareuttle.
	1	4	tret.

32 0 16 neat weight

2. What is the neat weight of 42 cwt. of sugar, tare in the whole 2 cwt. 1 qr. 6 lb., and tret 4 lb. in 104, or $\frac{1}{26}$?

Ans. 38 cwt. 0 qr. 19 lb.

3. Bought 4 boxes of soap, each weighing 3 cwt. 1 qr. 27 lb. gross, tare 14 lb. per box, tret as before; what is the neat weight of the whole, and what will it cost, at 37s. 4d. per cwt.?

Ans. { 12 cwt. 3 qr. 22 lb.
£24 3s. 4d.

4. Bought 61803 lb. of oil, at £25 10s. per ton, tare 14 tons, tret 4 lb. in 104; what is the neat weight and value?

Ans. { 56700 lb.
£637 10s.

CASE V.—*To find the neat weight, when tare, tret, and cloff are allowed.*

RULE.—Deduct for tare and tret, as in the last case, and from this result subtract 2 lb. for every 3 cwt., that is the 168th part, and the remainder will be the neat weight required.

EXAMPLES.

1. What is the neat weight of 303 cwt 2 qr. 10 lb., tare 15 cwt. 4 lb, tret 4 lb. per 104 lb., cloff 2 lb. per 3 cwt.?

cwt.	qr.	lb.	
303	2	10	gross.
15	0	4	tare.

26)	288	2	6	
	11	0	11	tret.

168)	277	1	23	
	1	2	16	15 oz. cloff.

275	3	6	1	neat weight.
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2. What is the neat weight of 121 cwt. 1 qr. 12 lb. gross, tare 10 cwt. 1 qr. 16 lb., tret 4 lb. per 104 lb., and cloff 2 lb. per 3 cwt.?

Ans. 106 cwt. 6 lb. 14 oz.

3. What is the neat weight of 26 casks, each weighing 4 cwt. 2 qr. 7 lb., tare per cask 1 cwt. 1 qr., tret 4 lb. per 104 lb., and cloff as usual?

Ans. 82 cwt. 1 qr. 7 lb. 12½ oz.

VULGAR FRACTIONS.

A vulgar fraction, or part of any thing, is expressed by two numbers, one placed above the other, with a line between them; as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{15}{16}$, &c.

The bottom number is called the denominator, and denotes how many equal parts the unit, number, or quantity is to be divided into; and the top number, which is called the numerator, denotes how many of these equal parts are to be taken; that is, if $\frac{3}{4}$ of a unit or quantity be required; here the denominator 4, denotes that the unit or quantity is to be divided into 4 equal parts, and the numerator 3 denotes that 3 of these equal parts are to be taken, and so on for any other fraction.

If the numerator and denominator be the same, the fraction is equal to one, as $\frac{4}{4} = 1$.

If the numerator be greater than the denominator, the fraction is greater than one, as $\frac{5}{4} = 1\frac{1}{4}$.

There are five sorts of vulgar fractions, viz. the proper, the improper, the compound, the mixed, and the complex.

1. A proper fraction is when the numerator is less than the denominator, as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{15}{16}$, &c.

2. An improper fraction is, when the numerator is equal to, or greater than the denominator, as $\frac{4}{4}$, $\frac{5}{4}$, $\frac{15}{3}$, &c.

3. A compound fraction is a fraction of a fraction, and known by the word *of* placed between them, as $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{6}{7}$, &c.

4. A mixed number is composed of a whole number and a fraction, as $2\frac{1}{2}$, $3\frac{1}{4}$, $12\frac{3}{4}$, &c.

5. A complex fraction contains a fractional part in the numerator or denominator, or in both, as $\frac{2\frac{1}{4}}{4}$, $\frac{3}{4\frac{2}{3}}$, $\frac{4\frac{1}{2}}{5\frac{3}{4}}$, $\frac{\frac{3}{4}}{\frac{6}{7}}$, &c.

NOTE.—A whole number may be expressed as a fraction by placing 1 for its denominator, as $4 = \frac{4}{1}$. The proper and improper fractions are called simple fractions.

CASE I.—*To reduce a given vulgar fraction to its lowest terms.*

RULE.—Divide the greater number by the less, and this divisor by the last remainder, and so on, till nothing remains; the last divisor will be the greatest common measure, or the greatest number, by which both the numerator and denominator can be divided without a remainder.

Divide the numerator and denominator of the given fraction by this common measure, and the quotients will be the numerator and denominator of the new fraction.

N. B. The value of a vulgar fraction is not changed by multiplying or dividing both its numerator and denominator by the same number.

EXAMPLES.

1. Reduce $\frac{125}{235}$ to its lowest terms.

$$\begin{array}{r}
 125 \overline{)235} 1 \\
 \underline{125} \\
 110 \\
 110 \overline{)125} 1 \\
 \underline{110} \\
 15 \\
 15 \overline{)110} 7 \\
 \underline{105} \\
 5 \\
 5 \overline{)15} 3 \\
 \underline{15}
 \end{array}$$

Then $5 \overline{) \frac{125}{235}} = \frac{25}{47}$ Ans.

- | | |
|---|----------------------|
| 2. Reduce $\frac{2\frac{3}{4}}{\frac{3}{2}}$ to its lowest terms. | Ans. $\frac{3}{2}$. |
| 3. Reduce $\frac{1\frac{9}{16}}{4\frac{8}{9}}$ to its lowest terms. | Ans. $\frac{3}{4}$. |
| 4. Reduce $\frac{1\frac{3}{7}}{2\frac{9}{5}}$ to its lowest terms. | Ans. $\frac{1}{5}$. |

- | | |
|---|-----------------------------|
| 5. Reduce $\frac{864}{891}$ to its lowest terms. | <i>Ans.</i> $\frac{5}{4}$ |
| 6. Reduce $\frac{1125}{1296}$ to its lowest terms. | <i>Ans.</i> $\frac{5}{6}$ |
| 7. Reduce $\frac{784}{896}$ to its lowest terms. | <i>Ans.</i> $\frac{7}{6}$ |
| 8. Reduce $\frac{1168}{2352}$ to its simplest form. | <i>Ans.</i> $\frac{4}{3}$ |
| 9. Reduce $\frac{342}{732}$ to its lowest terms. | <i>Ans.</i> $\frac{9}{6}$ |
| 10. Reduce $\frac{3285}{1926}$ to its lowest terms. | <i>Ans.</i> $\frac{9}{6}$ |
| 11. Reduce $\frac{1728}{1768}$ to its lowest terms. | <i>Ans.</i> $\frac{7}{16}$ |
| 12. Reduce $\frac{117819}{125664}$ to its lowest terms. | <i>Ans.</i> $\frac{15}{16}$ |
| 13. Simplify $\frac{1092}{182}$ as much as possible, | <i>Ans.</i> $\frac{3}{5}$ |

CASE II.—To reduce a mixed number to an improper fraction.

RULE.—Multiply the whole number by the denominator of the fractional part, to the product add the numerator, and place the denominator under the sum, for the improper fraction required.

EXAMPLES.

1. Reduce $12\frac{7}{8}$ to an improper fraction.

$$\begin{array}{r}
 12 \\
 8 \\
 \hline
 96 \\
 7 \\
 \hline
 103 \\
 \hline
 \text{Ans.} \\
 8
 \end{array}$$

- | | |
|--|----------------------------------|
| 2. Reduce $3\frac{1}{2}$ to an improper fraction. | <i>Ans.</i> $\frac{7}{2}$ |
| 3. Reduce $15\frac{5}{6}$ to an improper fraction. | <i>Ans.</i> $\frac{95}{6}$ |
| 4. Reduce $125\frac{1}{2}$ to an improper fraction. | <i>Ans.</i> $125\frac{1}{2}$ |
| 5. Reduce $374\frac{7}{8}$ to an improper fraction. | <i>Ans.</i> $299\frac{7}{8}$ |
| 6. Reduce $56\frac{9}{11}$ to an improper fraction. | <i>Ans.</i> $409\frac{9}{11}$ |
| 7. Place 4 ones in such a manner that they may be equal to 12. | <i>Ans.</i> $11\frac{1}{2} = 12$ |
| 8. Place 4 nines so that they may be equal to 1, 20, 100, 111, or 0. | |

CASE III.—To reduce an improper fraction to its whole equivalent or mixed number.

RULE.—Divide the numerator by the denominator, and the quotient will be the whole number ; if there be a remainder, place it on the right hand of the quotient, with the divisor below it for a denominator, as in Example 2nd.

EXAMPLES.

1. Reduce
- $7\frac{1}{2}$
- to its equivalent whole or mixed number.

$$\begin{array}{r} 12 \overline{)72} \\ \hline \end{array}$$

6 Ans.

2. Reduce
- $10\frac{3}{8}$
- to its equivalent whole or mixed number.

$$\begin{array}{r} 8 \overline{)103} \\ \hline \end{array}$$

12 $\frac{7}{8}$ Ans.

3. Reduce
- $15\frac{1}{2}$
- to its equivalent whole or mixed number.

Ans. 125 $\frac{1}{2}$.

4. Reduce
- $40\frac{6}{7}$
- to its equivalent whole or mixed number.

Ans. 56 $\frac{2}{7}$.

5. Reduce
- $17\frac{2}{3}$
- to its equivalent whole or mixed number.

Ans. 48.

6. Reduce
- $17\frac{4}{7}$
- to its equivalent whole or mixed number.

Ans. 23 $\frac{1}{7}$.

7. Reduce
- $7\frac{4}{5}$
- to its equivalent whole or mixed number.

Ans. 21 $\frac{7}{5}$.

8. Reduce
- $4\frac{6}{7}$
- to its equivalent whole or mixed number.

Ans. 20 $\frac{2}{7}$.

9. Simplify
- $15\frac{0}{5}$
- as much as possible.

Ans. 100 $\frac{1}{5}$.

CASE IV.—To reduce a compound fraction to a single one.

RULE.—Multiply all the numerators in the compound fraction together for a new numerator, and all the denominators for a new denominator, which will be the single fraction required—reduce this to its lowest terms.

Or if two or more factors be found both in the numerator and denominator equal to each other, they may be struck out; as also, if two factors, one in the numerator and the other in the denominator, have a common divisor, they may be divided by it: multiply what remains together as in the former part of the rule.

N. B. reduce the given fractions if necessary by the former rules as in example 2nd.

EXAMPLES.

1. Reduce
- $\frac{2}{3}$
- of
- $\frac{3}{4}$
- of
- $\frac{4}{5}$
- , to a single fraction.

$$\text{Here } \frac{2 \times 3 \times 4}{3 \times 4 \times 5} = \frac{24}{60} = \frac{2}{5} \text{ Ans.}$$

$$\text{Or } \frac{2 \times \cancel{3} \times \cancel{4}}{\cancel{3} \times \cancel{4} \times 5} = \frac{2}{5} \text{ Ans.}$$

2. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $4\frac{1}{2}$ of $\frac{9}{28}$ to a single fraction.

Here $4\frac{1}{2} = \frac{9}{2}$.

$$\text{Then } \frac{2 \times 7 \times 24 \times 9}{3 \times 8 \times 5 \times 28} = \frac{3024}{3360} = \frac{9}{10} \text{ Ans.}$$

$$\text{Or } \frac{\cancel{2} \times \cancel{7} \times \cancel{24} \times 9}{3 \times \cancel{8} \times 5 \times \cancel{28}_2} = \frac{9}{10} \text{ Ans.}$$

NOTE.—In the 1st example, the threes cut each other; as also, do the fours.

In the 2nd example, the product of the 3 and 8, in the denominator is 24, this will cancel the 24 in the numerator; and the product of the 2 and 7 in the numerator, is contained twice in the 28, in the denominator.

N. B. The following remark may tend to simplify this most important rule. If we take, for example, the compound fraction $\frac{3}{4}$ of $\frac{7}{8}$: this signifies that the magnitude or quantity denoted by $\frac{7}{8}$, is to be divided into 3 equal parts, one of which is $\frac{1}{8}$, and 2 of these equal parts to be taken; that is, $\frac{1}{8} \times 2 = \frac{2}{8}$; or thus $\frac{2}{8}$ of $\frac{3}{5} = \frac{2}{5}$ which is two-thirds of $\frac{3}{5}$.

This method of canceling the figures, very materially shortens the different operations in arithmetic, particularly in the Rule of Three and Compound Proportion: it is also one of the grand secrets in the German Chain Rule; therefore, the pupil ought to be well versed in its operations.

3. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ to a single fraction. Ans. $\frac{21}{32}$

4. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{9}{10}$ to a single fraction. Ans. $\frac{207}{1280}$

5. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{11}{16}$ to a single fraction. Ans. $\frac{207}{2048}$

6. Reduce $\frac{3}{4}$ of $2\frac{1}{2}$ of $\frac{1}{13}$ to a single fraction. Ans. $\frac{3}{104}$

7. Reduce $\frac{3}{4}$ of $7\frac{1}{2}$ of $2\frac{1}{13}$ to a single fraction. Ans. $\frac{1}{4}$ or $\frac{1}{4}$

8. Reduce $\frac{3}{4}$ of $4\frac{1}{2}$ of $2\frac{1}{13}$ of $5\frac{1}{2}$ to a single fraction. Ans. $\frac{3}{13}$

9. A person has $\frac{3}{4}$ of a mill, and sells $\frac{1}{2}$ of his share; what part of the mill does he dispose of? Ans. $\frac{3}{8}$

10. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{1}{2}$ of $\frac{9}{10}$ of $\frac{1}{7}$ to a single fraction. Ans. $\frac{1}{1120}$

11. Reduce $\frac{3}{4}$ of $1\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{2}$ to a simple fraction. Ans. $\frac{1}{2}$

CASE V.—To find the least common multiple of two or more numbers, that is, the least number that can be divided by the two or more numbers separately without a remainder.

RULE.—Place the given numbers as a dividend; which

divide by any number that will divide the most without a remainder, write down the quotients and the undivided numbers; (the undivided numbers are these which cannot be divided by the divisor without a remainder,) divide these and the succeeding results, in like manner till no devisable number, excepting by itself, remains. Multiply the several divisors and the undivisable numbers together, and their product will be the least common multiple required.

N. B. if no two numbers be devisable by the same number then the product of the given numbers will be the least common multiple as in example 5.

EXAMPLES.

1. What is the least common multiple of 2, 3, 4, 6, 9, 12, and 18?

$$\begin{array}{r}
 2)2, 3, 4, 6, 9, 12, 18 \\
 \hline
 3)1, 3, 2, 3, 9, 6, 9 \\
 \hline
 2)1, 2, 1, 3, 2, 3 \\
 \hline
 3)1, 3, 1, 3 \\
 \hline
 1, 1
 \end{array}$$

Then $2 \times 3 \times 2 \times 3 = 36$ the least common multiple or number which can be divided by 2, 3, 4, 6, 9, 12 and 18 without a remainder.

2. What is the least common multiple of 4, 5, 6, and 12?

$$\begin{array}{r}
 3)4, 5, 6, 12 \\
 \hline
 2)4, 5, 2, 4 \\
 \hline
 2)2, 5, 1, 2 \\
 \hline
 1, 5, 1
 \end{array}$$

Then $3 \times 2 \times 2 \times 5 = 60$ the least common multiple required.

3. What is the least common multiple of 4, 7, 8, 14, and 28? *Ans.* 56.

4. What is the least common multiple of 3, 5, 7, 21, and 35? *Ans.* 105.

5. Find the least common multiple of 3, 7, 8, and 11.

Ans. 1848.

6. What is the least number that can be divided by 4, 9, 12, 18, 36, or 8 without any remainder? *Ans.* 72.

CASE VI.—*To reduce fractions to a common denominator; that is, two or more fractions having different denominators, to reduce them to others of the same value, each having the same denominator.*

RULE 1st.—If any of the given quantities be mixed numbers, compound fractions, &c. reduce them to simple fractions by the former rules.

Multiply each numerator into all the denominators, excepting its own, for the new numerators, and all the denominators together for a common denominator.

EXAMPLES.

1. Reduce $\frac{2}{3}$, $\frac{1}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ to fractions of the same value, having a common denominator.

$$\begin{array}{rcl} 2 \times 4 \times 5 \times 6 = 240 & \text{the new numerator for } \frac{2}{3} \\ 1 \times 3 \times 5 \times 6 = 90 & \text{ditto for } \frac{1}{4} \\ 4 \times 3 \times 4 \times 6 = 288 & \text{ditto for } \frac{4}{5} \\ 5 \times 3 \times 4 \times 5 = 300 & \text{ditto for } \frac{5}{6} \\ 3 \times 4 \times 5 \times 6 = 360 & \text{the common denominator.} \end{array}$$

Then $\frac{240}{360}$, $\frac{90}{360}$, $\frac{288}{360}$, and $\frac{300}{360}$ are the new equivalent fractions

Proof—here $\frac{240}{360}$ reduced to its lowest terms by case 1st = $\frac{2}{3}$
 $\frac{90}{360}$ ditto = $\frac{1}{4}$
 $\frac{288}{360}$ ditto = $\frac{4}{5}$
 and $\frac{300}{360}$ ditto = $\frac{5}{6}$

Or thus

$$\begin{array}{rcl} \frac{2}{3} & = \frac{2 \times 4 \times 5 \times 6}{3 \times 4 \times 5 \times 6} = \frac{240}{360} \\ \frac{1}{4} & = \frac{1 \times 3 \times 5 \times 6}{3 \times 4 \times 5 \times 6} = \frac{90}{360} \\ \frac{4}{5} & = \frac{4 \times 3 \times 4 \times 6}{3 \times 4 \times 5 \times 6} = \frac{288}{360} \\ \frac{5}{6} & = \frac{5 \times 3 \times 4 \times 5}{3 \times 4 \times 5 \times 6} = \frac{300}{360} \end{array}$$

2. Reduce $\frac{3}{4}$, $3\frac{1}{2}$, $\frac{1}{2}$ of $\frac{5}{6}$, and 4 to simple fractions of the same value, having a common denominator.

Here $3\frac{1}{2} = \frac{7}{2}$, $\frac{1}{2}$ of $\frac{5}{6} = \frac{5}{12}$ and $4 = \frac{4}{1}$.

Therefore, the given question becomes $\frac{3}{4}$, $\frac{7}{2}$, $\frac{5}{12}$, and $\frac{4}{1}$.

$$\begin{aligned}
 2 \times 2 \times 3 \times 1 &= 12, \text{ new numerator for } \frac{1}{3} \\
 7 \times 3 \times 3 \times 1 &= 63, \text{ ---ditto--- for } \frac{1}{7} \\
 2 \times 3 \times 2 \times 1 &= 12, \text{ ---ditto--- for } \frac{1}{2} \\
 4 \times 3 \times 2 \times 3 &= 72, \text{ ---ditto--- for } \frac{1}{4} \\
 3 \times 2 \times 3 \times 1 &= 18, \text{ common denominator.}
 \end{aligned}$$

Then $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{2}$, and $\frac{1}{4}$ are the new equivalent fractions required, and may be proved by Case I.

3. Reduce $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{2}$ to fractions of the same value, having a common denominator. *Ans.* $\frac{2}{12}$, $\frac{9}{63}$, and $\frac{6}{12}$.

4. Reduce $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{7}$ to fractions of the same value, having a common denominator. *Ans.* $\frac{3}{12}$, $\frac{4}{12}$, $\frac{6}{12}$, and $\frac{6}{42}$.

5. Reduce $\frac{1}{3}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{1}{3}$, and $1\frac{1}{2}$ to simple fractions of the same value, having a common denominator. *Ans.* $\frac{1}{6}$, $\frac{1}{6}$, and $\frac{13}{6}$.

6. Find fractions equal in value to $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{10}$, having a common denominator. *Ans.* $\frac{10}{30}$, $\frac{42}{420}$, and $\frac{3}{30}$.

7. Find fractions equal in value to $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{4}$, having a common denominator. *Ans.* $\frac{48}{336}$, $\frac{42}{336}$, $\frac{112}{336}$, and $\frac{84}{336}$.

8. Reduce $\frac{1}{3}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $3\frac{1}{2}$, and $\frac{1}{7}$ of $9\frac{1}{2}$ to simple fractions having a common denominator. *Ans.* $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{13}{4}$.

9. Reduce $\frac{1}{3}$ of $1\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{2}$ of $4\frac{1}{2}$ to simple fractions, having a common denominator. *Ans.* $\frac{1}{12}$ and $\frac{1}{12}$.

10. Reduce $\frac{1}{4}$ of $3\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{2}$ of $2\frac{1}{2}$ of $\frac{1}{3}$ and $1\frac{1}{2}$ to equivalent fractions having a common denominator. *Ans.* $\frac{1}{12}$, $\frac{1}{12}$, and $\frac{1}{12}$.

RULE 2.—If it be required to reduce the given fractions to others of the same value, having the least common denominator.

Reduce the given fractions to their lowest terms; and find by Case V. the least common multiple of their denominators. Multiply the numerator and denominator of each of the given fractions by this common multiple, in the shape of a compound fraction; observing to cut the common multiple in the numerator by the denominator of the given fraction.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{6}$ to fractions of the same value, having the least common denominator.

$$2)2, 4, 6$$

$$1, 2, 3$$

Hence $2 \times 2 \times 3 = 12$, the least common multiple of the denominators.

$$\text{Then } \frac{1 \times 1 \times 6}{2 \times 12} = \frac{6}{12} \text{ the new fraction} = \frac{1}{2}.$$

$$\frac{3 \times 17^3}{4 \times 12} = \frac{9}{12} \text{ the new fraction} = \frac{3}{4}$$

$$\text{and } \frac{5 \times 17^2}{6 \times 12} = \frac{10}{12} \text{ the new fraction} = \frac{5}{6}$$

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to the least common denominator
Here $\frac{1}{4} = \frac{1}{4}$.

$$2) 2, 6, 4$$

$$\hline 1, 3, 2$$

Hence $2 \times 3 \times 2 = 12$, the least common multiple.

$$\text{Then } \frac{1 \times 17^6}{2 \times 12} = \frac{6}{12}, \text{ the new fraction} = \frac{1}{2}.$$

$$\frac{5 \times 17^2}{6 \times 12} = \frac{10}{12}, \text{ the new fraction} = \frac{5}{6}.$$

$$\text{and } \frac{1 \times 2^3}{4 \times 12} = \frac{3}{12}, \text{ the new fraction} = \frac{1}{4} \text{ or } \frac{3}{12}$$

3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions of the same value, having the least common denominator. *Ans.* $\frac{1}{60}$, $\frac{2}{60}$, $\frac{3}{60}$, and $\frac{4}{60}$

4. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions having the least common denominator. *Ans.* $\frac{1}{60}$, $\frac{2}{60}$, $\frac{3}{60}$, and $\frac{4}{60}$

5. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ to fractions of the same value, having the least common denominator.

$$\text{Ans. } \frac{1}{60}, \frac{2}{60}, \frac{3}{60}, \frac{4}{60}, \frac{5}{60}, \text{ and } \frac{6}{60}$$

6. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions of the same value, having the least common denominator.

$$\text{Ans. } \frac{1}{60}, \frac{2}{60}, \frac{3}{60}, \frac{4}{60}, \frac{5}{60}, \text{ and } \frac{6}{60}$$

7. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions of the same value, having the least common denominator. *Ans.* $\frac{1}{60}$, $\frac{2}{60}$, $\frac{3}{60}$, and $\frac{4}{60}$

8. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions of the same value, having the least common denominator. *Ans.* $\frac{1}{60}$, $\frac{2}{60}$, $\frac{3}{60}$, and $\frac{4}{60}$

9. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ to fractions of an equal value, having the least common denominator.

$$\text{Ans. } \frac{1}{420}, \frac{2}{420}, \frac{3}{420}, \frac{4}{420}, \frac{5}{420}, \text{ and } \frac{6}{420}$$

CASE VII.—To convert a whole number into a fraction, having any given denominator.

RULE.—Make the whole number into a fraction by placing one under it for a denominator; then multiply the numerator and denominator of this fraction by the proposed denominator. Or multiply and divide the given number by the given denominator.

EXAMPLES

1. Reduce 5 to a fraction, having 7 for its denominator.

Here $5 = \frac{5}{1}$; then $\frac{5 \times 7}{1 \times 7} = \frac{35}{7}$ Ans.

2. Reduce 7 to a fraction, having 8 for its denominator.

Ans. $\frac{56}{8}$.

3. Reduce 9 to a fraction, having 12 for its denominator.

Ans. $\frac{108}{12}$.

4. Reduce 2 to a fraction, having 5 to its denominator.

Ans. $\frac{10}{5}$.

CASE VIII.—To reduce a fraction to another of the same value, having a given numerator or denominator.

RULE.—If the numerator of the required fraction be given—state, as the numerator of the given fraction is to its denominator, so is the numerator of the required fraction to its denominator : as follows. Reduce $\frac{3}{4}$ to a fraction of the same value, having 6 for its numerator.

As 3 : 4 :: 6

4

—
3)24

—
8 new denominator.

Then $\frac{6}{8}$ is the new fraction required.

If the denominator of the required fraction be given—state, as the denominator of the given fraction is to its numerator, so is the denominator of the required fraction to its numerator : thus. Reduce $\frac{7}{8}$ to a fraction of the same value, having 24 for its denominator.

As 8 : 7 :: 24

7

—
8)168

—
21 the new numerator.

Then $\frac{21}{24}$ is the fraction required.

EXAMPLES.

1. Reduce $\frac{5}{6}$ to a fraction of the same value, having 12 for its numerator.

Ans. $\frac{12}{14\frac{2}{3}}$

2. Reduce $\frac{1}{5}$ to a fraction of the same value, having 25 for its denominator.

$$\text{Ans. } \frac{11\frac{1}{5}}{25}.$$

3. Reduce $\frac{1}{5}$ to a fraction of the same value, having 20 for its numerator.

$$\text{Ans. } \frac{20}{25}$$

4. Reduce $\frac{1}{5}$ to a fraction of the same value, having 56 for its denominator.

$$\text{Ans. } \frac{49}{56}$$

5. Reduce $\frac{11}{12}$ to a fraction of the same value, having 132 for its numerator.

$$\text{Ans. } \frac{132}{144}$$

6. Reduce $\frac{9}{10}$ to a fraction of the same value, having 30 for its denominator.

$$\text{Ans. } \frac{27}{30}$$

CASE IX.—*To reduce a complex fraction to a simple one of the same value.*

RULE.—As the value of a fraction is not changed by multiplying its numerator and denominator by the same number ; multiply the numerator and denominator of the given fraction by the denominator of the fractional part, as in Examples 1 and 2. But if the given fraction contain a fractional part in both its numerator and denominator, multiply the numerator and denominator of the given fraction by the common multiple of the denominators of the fractional parts, as in Example 3.

EXAMPLES.

1. Reduce $\frac{3\frac{1}{2}}{4}$ to its equivalent simple fraction.

$$\text{Here } \frac{3\frac{1}{2} \times 2}{4 \times 2} = \frac{7}{8} \text{ Ans.}$$

2. Reduce $\frac{2}{4\frac{1}{3}}$ to its equivalent simple fraction.

$$\text{Here } \frac{2 \times 3}{4\frac{1}{3} \times 3} = \frac{6}{13} \text{ Ans.}$$

3. Reduce $\frac{4\frac{2}{3}}{7\frac{2}{3}}$ to a simple fraction of the same value.

Here, by Case V. the least common multiple of 6 and 9, is 18.

$$\text{Then } \frac{4\frac{2}{3} \times 18}{7\frac{2}{3} \times 18} = \frac{87}{130} \text{ Ans.}$$

4. Reduce $\frac{2\frac{1}{2}}{4}$ to its equivalent simple fraction. *Ans.* $\frac{1}{2}$
5. Reduce $\frac{3}{4\frac{1}{2}}$ to its equivalent simple fraction. *Ans.* $\frac{2}{3}$
6. Reduce $\frac{3\frac{1}{2}}{5}$ to its equivalent simple fraction. *Ans.* $\frac{7}{10}$
7. Reduce $\frac{5}{6\frac{2}{3}}$ to a simple fraction of the same value. *Ans.* $\frac{3}{8}$
8. Reduce $\frac{4\frac{1}{2}}{5\frac{1}{2}}$ to its equivalent simple fraction. *Ans.* $\frac{9}{11}$
9. What simple fraction is equal in value to $\frac{4\frac{1}{2}}{7\frac{2}{3}}$? *Ans.* $\frac{14}{17}$
10. Reduce $\frac{2\frac{1}{2}}{4}$ of $\frac{3}{5}$ to a single fraction. *Ans.* $\frac{3}{10}$
11. Reduce $\frac{2\frac{1}{2}}{3\frac{1}{2}}$ of $\frac{6\frac{1}{2}}{15}$ to a simple fraction. *Ans.* $\frac{1}{3}$
12. Reduce $\frac{\frac{3}{4}}{4\frac{1}{2}}$ to a simple fraction. *Ans.* $\frac{3}{18} = \frac{1}{6}$
13. Reduce $\frac{2\frac{1}{2}}{4\frac{2}{3}}$ to a simple fraction. *Ans.* $\frac{5}{8}$
14. Reduce $\frac{4\frac{2}{3}}{6\frac{1}{3}}$ of $\frac{5}{9\frac{1}{3}}$ to a simple fraction. *Ans.* $\frac{2}{9}$
15. Reduce $\frac{2\frac{1}{2}}{4\frac{2}{3}}$ of $\frac{4\frac{1}{2}}{5}$ of $2\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{1}{2}$
16. Reduce $\frac{3\frac{7}{8}}{4\frac{5}{12}}$ to a simple fraction. *Ans.* $\frac{23}{18}$
17. Reduce $\frac{\frac{7}{8}}{4\frac{1}{2}}$ to a simple fraction. *Ans.* $\frac{7}{18}$
18. Reduce $\frac{4\frac{2}{3}}{6\frac{3}{4}}$ to a simple fraction. *Ans.* $\frac{16}{15}$
19. Reduce $\frac{\frac{2}{3}}{\frac{6}{7}}$ to a simple fraction, *Ans.* $\frac{7}{9}$
20. Reduce $\frac{\frac{17}{18}}{\frac{19}{18}}$ to a simple fraction. *Ans.* $\frac{17}{19}$

CASE X.—To find the value of a fraction in known parts of the integer.

RULE.—Multiply the given quantity or integer by the numerator of the fraction, and divide this product by the denominator, and the quotient will be the value required.

Or, divide the given quantity or integer by the denominator, and multiply the quotient by the numerator.

EXAMPLES.

1. What is the value of $\frac{3}{8}$ of a £. ?

$\begin{array}{r} 20s. \\ 3 \\ \hline 8)60 \\ \hline 7\ 6 \end{array}$	Or,	$\begin{array}{r} 8)20s. \\ \hline 2\ 6 \\ 3 \\ \hline 7\ 6 \end{array}$
<i>Ans.</i>		<i>Ans.</i>

2. What is the value of $\frac{1}{8}$ of 2 cwt. 2 qrs. 16 lb. ?

$\begin{array}{r} cwt. \ qr. \ lb. \\ 2 \ 2 \ 16 \\ 7 \\ \hline 8)18 \ 2 \ 0 \\ \hline 2 \ 1 \ 7 \end{array}$	Or,	$\begin{array}{r} cwt. \ qr. \ lb. \\ 8)2 \ 2 \ 16 \\ \hline 0 \ 1 \ 9 \\ 7 \\ \hline 2 \ 1 \ 7 \end{array}$
<i>Ans.</i>		<i>Ans.</i>

3. What is the value of $\frac{1}{4}$ of a guinea ? *Ans.* 12s.

4. What is the value of $\frac{1}{4}$ of a Cwt. ? *Ans.* 1 qr. 20 lb.

5. What is the value of $\frac{1}{4}$ of a lb. troy ? *Ans.* 9 oz. 12 dwt.

6. What is the value of $\frac{1}{8}$ of £3 12s. 6 $\frac{1}{2}$ d. ? *Ans.* £2 0s. 3 $\frac{1}{2}$ d.

7. Reduce $\frac{1}{4}$ of a cwt. to its proper quantity *Ans.* 3qr. 14lb.

8. What is the value of $\frac{1}{16}$ of a quarter of wheat ?
Ans. 7 bush. 2 pecks.

9. What is the value of $\frac{3}{4}$ of 4 yd. 2 qr. 3 n. ?
Ans. 2 yd. 3 qr. 1 n.

10. Find the value of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{8}$ of 100 guineas. *Ans.* £35

CASE XI.—To reduce fractions of one denomination to those of another, retaining the same value.

RULE I.—When the fraction is from a less name to a greater, multiply the denominator of the given fraction, by the several

denominations between it and the required one, for a new denominator, over which place the given numerator, as in Example 1st.

2. When the fraction is from a greater denomination to a less; multiply the given numerator by the several denominations, under which place the given denominator, as in Example 2nd.

3. When the given quantity is of several denominations; reduce it to the lowest denomination mentioned for a numerator, under which place the other quantity, reduced into the same denomination as the given quantity, for a denominator, as in Examples 3rd and 4th.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of a farthing to the fraction of a £. or into the denomination of a £.

$$\text{Here } \frac{2}{3 \times 4 \times 12 \times 20} = \frac{2}{2880} = \frac{1}{1440} \text{ £. Ans.}$$

2. Reduce $\frac{2}{7}$ of £. to the fraction of a farthing or to the denomination of farthings.

$$\text{Here } \frac{2 \times 20 \times 12 \times 4}{7 \times 1} = \frac{2880}{7} \text{ farthings, Ans.}$$

NOTE.—In the 1st Example, the given fraction, ($\frac{2}{3}$), is in the denomination of farthings; and, in order to bring farthings into pounds, we divide by 4, 12, and 20; or in other words, we multiply the denominator (3) by 4, 12, and 20.

In the 2nd Example, the given fraction, ($\frac{2}{7}$) is in the denomination of pounds; and to bring pounds into farthings, we multiply by 20, 12, and 4, or in other words, multiply the numerator (2) by 20, 12, and 4.

3. Reduce 3s. 6½d. to the fraction of a £. or, what part of a £. is 3s. 6½d.? Or express 3s. 6½d. in the denomination of £s.

<i>s.</i>	<i>d.</i>	<i>s.</i>
3	6½	20
12		12
42		240
2		2
85		480

$$\text{Then } \frac{85}{480} = \frac{17}{96} \text{ £. Ans.}$$

4. Reduce 2 qrs. 14 lbs. $10\frac{1}{2}$ oz. to the fraction of a cwt.

<i>qrs.</i>	<i>lbs. oz.</i>	<i>lb.</i>
2	14 10½	112
28		16
70		1792
16		5
1130		
5		8960
5653		

Then $\frac{5653}{8960}$ cwt. *Ans.*

5. What part of a £. is $\frac{1}{2}$ of a shilling? *Ans.* $\frac{1}{100}$ £.
6. Reduce $\frac{1}{2}$ of a penny to the fraction of a £. *Ans.* $\frac{1}{200}$ £.
7. Reduce $\frac{1}{2}$ of a shilling to the fraction of a moidore. *Ans.* $\frac{1}{16}$ m.
8. Reduce $\frac{1}{2}$ of a crown to the fraction of a £. *Ans.* $\frac{1}{4}$ £.
9. What part of a cwt. is $\frac{1}{2}$ of a lb.? *Ans.* $\frac{1}{168}$ cwt.
10. Reduce 1 cwt. 2 qrs. 14 lb. to the fraction a cwt. *Ans.* $1\frac{1}{2}$ or $\frac{3}{2}$ cwt.
11. Reduce $\frac{1}{2}$ of a grain to the fraction of a lb troy. *Ans.* $\frac{1}{6912}$ lb.
12. Reduce $\frac{1}{2}$ of a lb. troy to the fraction of a grain. *Ans.* 17280
13. What part of a yard is $\frac{1}{2}$ of a Flemish ell? *Ans.* $\frac{1}{2}$ yd.
14. Reduce $\frac{1}{2}$ of a French ell to the fraction of a yard. *Ans.* $\frac{2}{3}$ yd.
15. Reduce $\frac{1}{17}$ of a mile to the fraction of a yard. *Ans.* $\frac{1740}{17}$ yards.
16. Express $7\frac{1}{2}$ yards in the denomination of a mile. *Ans.* $\frac{3}{160}$
17. Reduce $\frac{1}{8}$ of a gallon of ale to the fraction of an hhd. *Ans.* $\frac{1}{48}$
18. Reduce $\frac{1}{8}$ of a barrel of beer to the fraction of a gallon. *Ans.* $\frac{180}{8} = \frac{45}{2}$
19. What part of a quarter is $\frac{1}{2}$ of a bushel? *Ans.* $\frac{1}{72}$
20. Reduce $\frac{1}{4}$ of a perch into acres. *Ans.* $\frac{1}{240}$
21. Reduce $\frac{1}{25}$ of an acre to the fraction of a perch. *Ans.* $\frac{16}{25}$
22. Reduce $\frac{1}{2}$ of a day to the fraction of a year, or $365\frac{1}{4}$ days. *Ans.* $\frac{1}{730}$
23. Reduce $\frac{1}{2}$ of a yard to the fraction of an English ell. *Ans.* $\frac{1}{3}$

24. What part of a £. is 3s. 4½d ? *Ans.* $\frac{163}{960}$
25. What part of 2 miles is 1 mile 6 fur. 16 poles ? *Ans.* $\frac{9}{16}$
26. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of 2 qrs. 14 lb. to the fraction of a cwt. *Ans.* $\frac{5}{16}$
27. Reduce 1 cwt. 2 qrs. 6 lb. 3 oz. 8½ dr. to the fraction of 3 cwt. *Ans.* $\frac{14}{17}$
28. Reduce $\frac{3}{5}$ of a £. into guineas. *Ans.* $\frac{4}{7}$
29. Reduce $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of a £. to the fraction of a moidore. *Ans.* $\frac{3}{35}$
30. Reduce $\frac{2\frac{1}{2}}{3\frac{1}{4}}$ of a £. into moidores. *Ans.* $\frac{33}{35}$
31. What part of a shilling is $\frac{3}{4}$ of 9d. ? *Ans.* $\frac{9}{16}$
32. What part of a £. is $\frac{2}{3}$ of $\frac{3}{4}$ of 7s. 6d. ? *Ans.* $\frac{3}{8}$
33. Reduce $\frac{3}{5}$ of $\frac{5}{8}$ of 4½ gallons of ale into the denomination of hogshheads. *Ans.* $\frac{1}{3}$ hhds.
34. Reduce 4 oz. 12 dr. avoirdupois to the fraction of a lb. troy. *Ans.* $\frac{323}{9216}$

ADDITION OF VULGAR FRACTIONS.

RULE.—Bring the given fractions, if required, into their most simple forms, and reduce them to a common denominator; add the new numerators into one sum, under which place the common denominator, and the result will be the sum required.

If the given fractions be of several denominations, find the value of each separately, and the sum of these results will be the answer required.

NOTE.—If the given fractions be mixed numbers, find the sum of the fractional parts, to which add the whole numbers, as in Example 3rd.

EXAMPLES.

1. Required the sum of $\frac{2}{3}$ and $\frac{1}{4}$.

$$\begin{array}{rcl} 2 \times 4 = 8 \\ 3 \times 3 = 9 \\ 3 \times 4 = 12 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{new numerators.} \\ \text{common denominator.} \end{array}$$

Then $\frac{2}{3} + \frac{1}{4} = \frac{11}{12} = 1\frac{1}{12}$, sum required.

What is the sum of $\frac{3}{8}$, $\frac{1}{5}$ of $\frac{1}{8}$, and $12\frac{1}{4}$?

$$\text{Here } \frac{3}{8} \text{ of } \frac{1}{8} = \frac{3}{8}$$

$$\left. \begin{array}{l} 2 \times 8 \times 4 = 64 \\ 3 \times 3 \times 4 = 36 \\ 3 \times 3 \times 8 = 72 \end{array} \right\} \text{new numerators.}$$

$$3 \times 8 \times 4 = 96 \text{ common denominator.}$$

$$\text{Then } \frac{64}{96} + \frac{36}{96} + 12 \frac{72}{96} = 12 \frac{172}{96} = 13 \frac{1}{24}.$$

Or thus,

Here, by Case 5, 24 is the least common multiple of 3, 8 and 4. Then, by Rule 2, Case 6, $\frac{3}{8} = \frac{1}{24}$, $\frac{1}{5}$ of $\frac{1}{8} = \frac{1}{24}$, and $12\frac{1}{4} = 12\frac{6}{24}$. Therefore $\frac{1}{24} + \frac{1}{24} + 12\frac{6}{24} = 12\frac{8}{24} = 13\frac{1}{3}$ Ans.

3. Required the sum of $14\frac{1}{2}$, $27\frac{2}{3}$, $9\frac{1}{4}$, and $15\frac{1}{3}$.

$$\left. \begin{array}{l} 3 \times 6 \times 12 \times 3 = 648 \\ 5 \times 4 \times 12 \times 3 = 720 \\ 1 \times 4 \times 6 \times 3 = 72 \\ 2 \times 4 \times 6 \times 12 = 576 \\ 4 \times 6 \times 12 \times 3 = 864 \end{array} \right\} \begin{array}{l} \text{new numerators} \\ \text{common denominator.} \end{array}$$

$$\text{Then, } 14\frac{648}{648} + 27\frac{720}{648} + 9\frac{72}{648} + 15\frac{576}{648} = 65\frac{16}{648} = 67\frac{1}{3}, \text{ Ans.}$$

Or thus,

Here, by Case 5, 12 is the least common multiple of 4, 6, 12, and 3.

$$\text{Then, } \frac{1}{2} = \frac{6}{12}, \frac{2}{3} = \frac{8}{12}, \text{ and } \frac{1}{3} = \frac{4}{12}.$$

$$\text{Therefore, } 14\frac{6}{12} + 27\frac{8}{12} + 9\frac{3}{12} + 15\frac{4}{12} = 65\frac{21}{12} = 67\frac{1}{3}, \text{ Ans.}$$

4. Add $\frac{2}{3}$ and $\frac{1}{5}$ together.

$$\text{Ans. } \frac{10}{15} = 1\frac{1}{3}$$

5. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?

$$\text{Ans. } 1\frac{1}{12}$$

6. Required the sum of $2\frac{1}{3}$ and $4\frac{1}{2}$.

$$\text{Ans. } 7\frac{1}{2}$$

7. What is the sum of $\frac{2}{3}$, $\frac{1}{5}$, $1\frac{4}{5}$, and $\frac{1}{6}$?

$$\text{Ans. } 2\frac{11}{30}$$

8. Add $\frac{2}{3}$ of $\frac{1}{4}$ and $\frac{1}{5}$ of $\frac{1}{6}$ together.

$$\text{Ans. } \frac{7}{6} = 1\frac{1}{6}$$

9. Add $2\frac{1}{2}$, $3\frac{1}{3}$, and $4\frac{1}{4}$ together.

$$\text{Ans. } 10\frac{1}{12}$$

10. Add $\frac{2}{3}$ of $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{1}{5}$, and 4 together.

$$\text{Ans. } 5\frac{1}{15}$$

11. Add $2\frac{1}{3}$, $3\frac{1}{4}$, and $7\frac{1}{8}$ together.

$$\text{Ans. } 14\frac{1}{4}$$

12. Add $\frac{2}{3}$ of $2\frac{1}{2}$, $\frac{1}{4}$ of $2\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{4}$ together.

$$\text{Ans. } 3\frac{7}{12}$$

13. Add $\frac{2}{3}$ of a £. $\frac{1}{8}$ of a shilling and $\frac{1}{4}$ of 9d. together.

$$\text{Ans. } 14s. 3\frac{1}{2}d.$$

14. Add $3\frac{1}{4}$ £. $2\frac{1}{8}$ s, and $\frac{1}{3}$ of $\frac{1}{4}$ of 6d. together.

$$\text{Ans. } £3 13s.$$

15. Add $\frac{2}{3}$ of £100, $\frac{1}{4}$ of 15s. 5d. and $\frac{1}{5}$ of half a crown together.

$$\text{Ans. } £67 7s. 2d.$$

16. Add $\frac{1}{8}$ of a cwt. $\frac{1}{4}$ of 2 qrs. and $3\frac{1}{2}$ lb. together.

$$\text{Ans. } 1 \text{ cwt. } 13 \text{ lb. } 10\frac{1}{2} \text{ oz.}$$

17. Add $\frac{3}{4}$ of a lb. $\frac{3}{4}$ of an oz. and $\frac{3}{4}$ of a dwt. together.
Ans. 8 oz. 12 dwt. 9 grs.
18. Add $\frac{3}{4}$ of a yard, $\frac{1}{2}$ of a foot, and $\frac{1}{4}$ of a mile together.
Ans. 1540 yds. 2 ft. 10 in.
19. Add $\frac{7}{8}$ of a French ell, $\frac{7}{10}$ of an English ell, $\frac{2}{3}$ of a yard, and $\frac{1}{5}$ of a Flemish ell together.
Ans. 3 yds. 1 qr. 33 n.
20. Add $\frac{3}{4}$ of $\frac{3}{4}$ of £350 16s. 3d., $\frac{4}{5}$ of 7 guineas, $\frac{3}{7}$ of $\frac{7}{8}$ of 24 quarter guineas, and $\frac{4}{5}$ of 36 moidores together.
Ans. £167 11s. 1d.
21. Add $3\frac{1}{2}$ £. $4\frac{1}{2}$ moidores, $\frac{1}{2}$ of a shilling, and $\frac{3}{4}$ of a penny together.
Ans. £9. 19s. 8 $\frac{1}{2}$ d.
22. What is the value of $\frac{2}{3}$ of £36 17s. 6d., and $\frac{3}{4}$ of $\frac{3}{4}$ of 28 quarter guineas?
Ans. £10 5s. 10 $\frac{3}{4}$ d.
23. What is the value of £3 $\frac{3}{8}$, $\frac{2}{3}$ of $\frac{5}{7}$ of a shilling, and $\frac{3}{7}$ of a penny?
Ans. £3 18s. 1 $\frac{1}{2}$ d.
24. Add $\frac{2}{3}$ of $\frac{3}{4}$ of £72 12s 7 $\frac{3}{4}$ d., $\frac{2}{3}$ of $\frac{1}{2}$ of half a crown, $\frac{7}{8}$ of 320 quarter guineas, $\frac{4}{5}$ of a shilling, $\frac{1}{6}$ of 7 $\frac{1}{2}$ d. and $\frac{5}{12}$ of a penny together.
Ans. £94 6s. 11 $\frac{1}{12}$ d.
25. A flax mill worth £12350, a coal mine £3241, and a ship £6500; what is the worth of $\frac{2}{3}$ of $\frac{3}{4}$ of the mill, $\frac{1}{2}$ of $\frac{1}{3}$ of the mine, and $\frac{1}{3}$ of $\frac{1}{2}$ of the ship?
Ans. £7632 16s. 8 $\frac{1}{2}$ d.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.—Reduce the given fractions to a common denominator, and subtract the less numerator from the greater; under this remainder place the common denominator, and the result will be the difference required.

EXAMPLES.

1. What is the difference between $\frac{2}{3}$ and $\frac{1}{4}$?

$$\begin{array}{rcl} 2 \times 5 = 10 & & \\ 3 \times 3 = 9 & \left. \begin{array}{l} \\ \end{array} \right\} \text{new numerators.} & \\ 3 \times 5 = 15 & \text{common denominator.} & \end{array}$$

Then $\frac{10}{15} - \frac{9}{15} = \frac{1}{15}$, the difference.

2. What is the difference between $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{4}{5}$?

Here $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$ and $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{4}{5} = \frac{3}{10}$.

$$\begin{array}{rcl} 2 \times 9 = 18 & & \\ 2 \times 5 = 10 & \left. \begin{array}{l} \\ \end{array} \right\} \text{new numerators} & \\ 5 \times 9 = 45 & \text{common denominator.} & \end{array}$$

Then $\frac{18}{45} - \frac{10}{45} = \frac{8}{45}$, *Ans.*

3. Required the difference between $\frac{1}{2}$ and $\frac{2}{3}$. *Ans.* $\frac{1}{6}$
4. From $\frac{7}{8}$ take $\frac{2}{3}$. *Ans.* $\frac{1}{24}$
5. What is the difference between $\frac{4}{5}$ and $\frac{5}{6}$? *Ans.* $\frac{1}{30}$
6. What is the difference between $4\frac{3}{4}$ and $3\frac{1}{2}$? *Ans.* $1\frac{1}{4}$
7. Required the difference between $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$. *Ans.* $\frac{1}{4}$
8. Required the difference between $\frac{4}{5}$ of $\frac{5}{6}$ and $\frac{2}{3}$ of $\frac{4}{5}$. *Ans.* $\frac{1}{15}$
9. What is the difference between $\frac{1}{3}$ of $\frac{5}{6}$ and $\frac{2}{3}$ of $3\frac{1}{2}$? *Ans.* $1\frac{1}{6}$
10. What is the difference between $\frac{3}{4}$ of a £. and $\frac{1}{2}$ of a shilling? *Ans.* 12s. 5½d.
11. What is the difference between $\frac{2}{3}$ of £3. 12s. and $\frac{1}{3}$ of $\frac{1}{2}$ of a guinea? *Ans.* £1. 11s. 7½d.
12. What is the difference of $\frac{1}{3}$ of a cwt. and $\frac{1}{3}$ of a qr.? *Ans.* 2 qrs. 1 lb. 13½ oz.
13. What is the difference between $\frac{2}{3}$ of a cwt. and $\frac{1}{3}$ of a cwt.? *Ans.* 1 qr. 7 lb. 12½ oz.
14. What is the difference between $\frac{1}{3}$ of a £. and $\frac{1}{12}$ of $\frac{1}{12}$ of a guinea? *Ans.* 10½d. 11½
15. Bought 25 cwt. 2 qr. 14 lb. of sugar, and parted with $\frac{1}{3}$ of the same; how much had I left? *Ans.* 16 cwt. 1½ lb.
16. Bought a waggon load of hay, which weighed, including the waggon, 2 tons, 17 cwt. 3 qrs. Now the waggon weighed $\frac{1}{3}$ of the whole. Required the weight of the hay and waggon separately. *Ans.* $\left\{ \begin{array}{l} 1 \text{ T. } 4 \text{ cwt. } 3 \text{ qrs. wt. of waggon.} \\ 1 \text{ T. } 13 \text{ cwt. } \quad \quad \text{wt. of hay.} \end{array} \right.$
17. What is the difference between $\frac{1}{3}$ of 7 yards and $\frac{1}{3}$ of an English ell? *Ans.* 3 yds. 1½ qr.
18. What is the difference between $\frac{1}{3}$ of an English ell and $\frac{1}{3}$ of a Flemish ell? *Ans.* $\frac{2}{3}$ yds.
19. Required the sum and difference of $\frac{1}{3}$ and $\frac{2}{3}$. *Ans.* $1\frac{1}{3}$ sum, $\frac{1}{3}$ difference.
20. What is the sum and difference of $\frac{1}{3}$ of $\frac{1}{3}$ and $\frac{2}{3}$ of $\frac{1}{3}$? *Ans.* $1\frac{2}{3}$ sum, $\frac{1}{3}$ difference.
21. What number added to $\frac{1}{3}$ will make $1\frac{1}{3}$? *Ans.* $1\frac{1}{3}$
22. A, B, and C, had a ship, $\frac{1}{3}$ belonged to A, and $\frac{1}{3}$ to B; what part of it belonged to C? *Ans.* $\frac{1}{3}$ C's.
23. What number added to $\frac{1}{3}$ of $\frac{1}{3}$ of $\frac{1}{3}$ will make 2? *Ans.* $1\frac{1}{3}$
24. A person had $\frac{1}{3}$ of a mill and sold $\frac{1}{3}$ of his share; how much had he left? *Ans.* $\frac{1}{3}$

25. Required the difference between $7 + \frac{1}{2}$ and $7 \times \frac{1}{2}$.

Ans. $2\frac{1}{2}$

26. Which is the greatest of the three fractions $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$?

Ans. $\frac{1}{3}$

27. It is required to simplify the following expression.

$$\frac{1}{2} \times \left\{ 100 - \frac{1}{3} \text{ of } 100\frac{1}{2} + \frac{7\frac{1}{2}}{2\frac{1}{4}} \right\}$$

Ans. $26\frac{26}{89}$

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.—Reduce the fractions, if required, to simple ones; then multiply the numerators together for a numerator, and the denominators for a denominator, and this new fraction will be the product required.

This rule is only the application of Case 4th.

To multiply a vulgar fraction by a whole number, multiply the numerator only, and retain the given denominator.

EXAMPLES.

1. Multiply $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$ and $\frac{7}{8}$ together.

$$\text{Here } \frac{4 \times 5 \times 6 \times 7}{5 \times 6 \times 7 \times 8} = \frac{840}{1680} = \frac{1}{2} \text{ Ans.}$$

$$\text{Or, } \frac{4 \times 5 \times 6 \times 7}{5 \times 6 \times 7 \times 8} = \frac{1}{2}, \text{ Ans.}$$

2. What is the product of $\frac{2}{3}$, $\frac{1}{4}$, $4\frac{1}{2}$, and $\frac{9}{28}$? *Ans.* $\frac{18}{14}$

NOTE.—The solution to this question is the same as question 2, Case 4.

3. Multiply $\frac{3}{5}$ by $\frac{5}{8}$ *Ans.* $\frac{3}{8}$

4. What is the continued product of $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$? *Ans.* $\frac{4}{7}$

5. Find the product of $\frac{3}{4}$, $2\frac{1}{2}$, and $\frac{4}{15}$. *Ans.* $\frac{1}{3}$

6. Multiply $\frac{7}{12}$, $7\frac{1}{3}$, and $2\frac{1}{12}$ together. *Ans.* 7.

7. Multiply $\frac{9}{25}$, $4\frac{1}{3}$, $2\frac{7}{18}$, and $5\frac{1}{2}$ together. *Ans.* $\frac{37}{5} = 9\frac{2}{5}$

8. Multiply $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{4}{5}$ of $1\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{3}$ together. *Ans.* $\frac{2}{5}$

9. Multiply 2s. 6d. by 3s. 6d. a shilling being the integer.

Ans. 8s. 9d.

10. Multiply 17s. 6d. by 13s. 4d., a pound being the integer.

Ans. $\frac{7}{11}$ £.

11. Multiply £99 19s. 11½d. by itself, a pound being the integer, or having the product in £. *Ans.* £9999 15s. 10½d.

12. Multiply 2 cwt. 3 qr. by 2 qr. 14 lb., a cwt. being the integer. *Ans.* 1 cwt. 2 qr. 24 lb. 8 oz.

13. What is the sum, difference, and product of $\frac{1}{2}$ and $\frac{1}{3}$?
Ans. $1\frac{1}{6}$ sum, $\frac{1}{6}$ difference, $\frac{1}{6}$ product.

14. A shilling weighs 3 dwts. 15 grs., of which 3 parts out of 40 are alloy, and the rest pure silver. How much per cent. is there of alloy, and what weight of pure silver?

Ans. $7\frac{1}{2}$ per cent., 3 dwts. $8\frac{1}{2}$ grs. pure silver

DIVISION OF VULGAR FRACTIONS.

RULE—(a) Invert the diviser, and proceed as in multiplication.

To divide a fraction by a whole number. Multiply the denominator of the fraction by the whole number, and retain the same numerator.

EXAMPLES.

1. Divide $\frac{4}{5}$ by $\frac{7}{8}$

$$\text{Here } \frac{4}{5} \div \frac{7}{8} = \frac{4 \times 8}{5 \times 7} = \frac{32}{35} \text{ } \textit{Ans.}$$

(a) **DEMONSTRATION.**—To divide one fraction by another, is to determine what multiple, part, or parts, the former is of the latter:—Thus, $\frac{m}{n} \div \frac{r}{s}$, proposes to determine what multiple, part, or parts, $\frac{m}{n}$ is of $\frac{r}{s}$;

that is, what multiple part or parts we must take of $\frac{r}{s}$ to produce $\frac{m}{n}$. Now,

$\frac{m}{n} \div \frac{r}{s}$, is s times $\frac{m}{n} \div r$; because the divisor is, in this case, s times too large; that is, $\frac{m}{n} \div \frac{r}{s} = \frac{m}{nr} \times s = \frac{ms}{nr} = \frac{m}{n} \times \frac{s}{r} \dots (1)$

In words, to divide one fraction by another, we must invert the dividing fraction, and proceed as in multiplication. From (1), $\frac{m}{n} \div \frac{r}{s} = \frac{ms}{nr}$;

whence, Definition, this signifies only that the $\left(\frac{ms}{nr}\right)^{th}$ part of $\frac{r}{s} = \frac{m}{n}$,

or $\frac{r}{s} \times \frac{m}{nr} = \frac{m}{n}$. And as $\frac{r}{s} \times \frac{ms}{nr} = \frac{ms}{nr} \times \frac{r}{s} = \frac{m}{n}$, we

conclude, that to divide $\frac{m}{n}$ by $\frac{r}{s}$, we have only to find such a quantity,

that the $\left(\frac{r}{s}\right)^{th}$ part of, shall be equal to $\frac{m}{n}$.

2. Divide $\frac{3}{7}$ of $\frac{3}{7}$ by $\frac{1\frac{1}{2}}{2\frac{1}{2}}$

$$\text{Here } \frac{2 \times 3}{3 \times 7} = \frac{2}{7} \text{ and } \frac{1\frac{1}{2} \times 6}{2\frac{1}{2} \times 6} = \frac{9}{14}$$

$$\text{Then } \frac{2 \times \cancel{14}^2}{\cancel{7} \times 9} = \frac{4}{9} \text{ Ans.}$$

$$\text{Or thus } \frac{2 \times 3 \times \cancel{2}^1}{3 \times \cancel{7}^1 \times 1\frac{1}{2}} = \frac{2}{4\frac{1}{2}} = \frac{4}{9} \text{ Ans.}$$

3. Divide $\frac{3}{7}$ by $\frac{1}{4}$.

$$\text{Ans. } \frac{4}{7} = 1\frac{1}{7}$$

4. Divide $\frac{1}{4}$ by $\frac{3}{4}$.

$$\text{Ans. } \frac{10}{15} = 1\frac{1}{3}$$

5. Divide $\frac{3}{7}$ by $\frac{4}{7}$.

$$\text{Ans. } \frac{3}{4}$$

6. Divide $\frac{3}{7}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{6}{7}$.

$$\text{Ans. } \frac{3}{4}$$

7. Divide $\frac{1}{4}$ of $\frac{6}{7}$ by $1\frac{1}{4}$.

$$\text{Ans. } \frac{8}{7}$$

8. Divide $\frac{3}{7}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{6}{7}$.

$$\text{Ans. } \frac{3}{4}$$

9. Divide $3\frac{1}{4}$ by $2\frac{1}{4}$.

$$\text{Ans. } 1\frac{1}{4}$$

10. Divide $4\frac{3}{4}$ by $3\frac{3}{4}$.

$$\text{Ans. } 1\frac{2}{7}$$

11. Divide $\frac{2\frac{1}{2}}{3\frac{1}{4}}$ by $\frac{6\frac{3}{4}}{10}$

$$\text{Ans. } 1\frac{1}{8}$$

12. What is the quotient of $\frac{1\frac{1}{2}}{2\frac{1}{2}}$ divided by $\frac{3}{7}$? *Ans. $\frac{3}{4} = 1\frac{1}{4}$*

13. Divide $\frac{2}{5}$ of $\frac{5}{6}$ of $3\frac{1}{2}$ by $\frac{4\frac{3}{4}}{5\frac{1}{4}}$.

$$\text{Ans. } 1\frac{11}{24}$$

14. Divide 3s. 6d. by 2s. 6d., a shilling being the integer, or having the quotient in shillings. *Ans. 1s. 4 $\frac{3}{4}$ d. $\frac{1}{2}$ of a farth.*

15. How many yards of cloth can I buy for £3 7s. 6d., at 8 $\frac{3}{4}$ shillings per yard? *Ans. 7 yds. 3 $\frac{1}{3}$ qr.*

16. In 7 cwt. 3 qrs. 14 lbs., how many parcels, each weighing 1 $\frac{1}{4}$ quarters? *Ans. 22 $\frac{10}{11}$*

17. Bought 25 cwt. 3 qr. 14 lb. of hay, which I made up into quantities, each containing 3 $\frac{1}{2}$ stones. How many had I? *Ans. 61 $\frac{1}{2}$*

18. What is the sum, difference, product, and quotient of $\frac{3}{8}$ and $\frac{1}{4}$? *Ans. $\frac{30}{80}$ sum, $\frac{2}{40}$ diff., $\frac{3}{20}$ prod., $1\frac{1}{2}$ quot.*

19. What is the sum, difference, product, and quotient of $\frac{1}{3}$ of $\frac{3}{4}$ and $\frac{1}{4}$? *Ans. $\frac{25}{48}$ sum, $\frac{1}{48}$ diff., $\frac{1}{48}$ prod., $1\frac{1}{2}$ quot.*

20 Required the sum, difference, product, and quotient of $\frac{2\frac{1}{2}}{3\frac{1}{2}}$ and $\frac{3}{5}$ of $\frac{5}{8}$ *Ans. $1\frac{1}{4}$ sum, $\frac{1}{4}$ differ., $\frac{2}{3}$ prod., 2 quot.*

21. Required the sum, difference, product, and quotient of $\frac{2\frac{1}{2}}{3\frac{1}{2}}$ and $\frac{3\frac{1}{4}}{4\frac{1}{2}}$ *Ans. $1\frac{57}{90}$ sum, $\frac{19}{90}$ diff., $\frac{91}{90}$ prod., $1\frac{13}{23}$ quot.*

22. What number multiplied by $\frac{4}{5}$ of $\frac{5}{6}$ will make $7\frac{7}{8}$?

Ans. $11\frac{13}{16}$

23. How many persons can you relieve, with $2\frac{3}{4}$ shillings each, out of £12 12s. $6\frac{1}{2}$ d.?

Ans. $106\frac{1}{3}$

24. Reduce $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

$$\begin{array}{r} \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \\ \hline \frac{15}{30} + \frac{10}{30} + \frac{7\frac{1}{2}}{30} + \frac{6}{30} \\ \hline \frac{38\frac{1}{2}}{30} \end{array}$$

to its simplest form.

25. Simplify $2\frac{4}{3} + \frac{3}{5}$

Ans. $1\frac{68}{157}$

$$\begin{array}{r} 2\frac{4}{3} + \frac{3}{5} \\ \hline 2\frac{20}{15} + \frac{9}{15} \\ \hline 2\frac{29}{15} \end{array}$$

Ans. $1\frac{82}{157}$

26. Divide 1 by a fraction, which has 1 for its numerator and 365 for its denominator.

Ans. 365

RULE OF THREE, DIRECT AND INVERSE IN VULGAR FRACTIONS.

RULE.—State the question, by Rule 2nd, in whole numbers; and reduce the given quantities, if required, to simple fractions, having the 1st and 2nd terms in the same denomination; then multiply the 2nd and 3rd terms together, and divide the product by the first, and the quotient will be the answer, in the same denomination as the third term is left in.

EXAMPLES.

1. Bought 3 cwt. 2 qrs. 14 lb. of sugar for £8 13s. 4d.; what is the value of $5\frac{3}{4}$ cwt. at the same rate?

cwt. qr. lb. cwt. £. s. d.

As 3 2 14 : $5\frac{3}{4}$:: 8 13 4

cwt. qr. lb. cwt. £. s. d.

Here 3 2 14 = $3\frac{1}{2}$ = $\frac{7}{2}$ cwt., 8 13 4 = $8\frac{1}{2}$ = $\frac{17}{2}$ £.
and $5\frac{3}{4}$ = $\frac{11}{2}$ cwt.

Then $\frac{8 \times 26 \times 17}{29 \times 3 \times 3} = \frac{3536}{261} = £13 \text{ } 10\text{s. } 11\frac{1}{2}\text{d. } \frac{1}{2}\text{p. } \frac{1}{4}\text{q. } \frac{1}{8}\text{r. } \frac{1}{16}\text{s. } \frac{1}{32}\text{d. } \frac{1}{64}\text{c. } \frac{1}{128}\text{m. } \frac{1}{256}\text{n. } \frac{1}{512}\text{o. } \frac{1}{1024}\text{p. } \frac{1}{2048}\text{q. } \frac{1}{4096}\text{r. } \frac{1}{8192}\text{s. } \frac{1}{16384}\text{t. } \frac{1}{32768}\text{u. } \frac{1}{65536}\text{v. } \frac{1}{131072}\text{w. } \frac{1}{262144}\text{x. } \frac{1}{524288}\text{y. } \frac{1}{1048576}\text{z. } \frac{1}{2097152}\text{aa. } \frac{1}{4194304}\text{ab. } \frac{1}{8388608}\text{ac. } \frac{1}{16777216}\text{ad. } \frac{1}{33554432}\text{ae. } \frac{1}{67108864}\text{af. } \frac{1}{134217728}\text{ag. } \frac{1}{268435456}\text{ah. } \frac{1}{536870912}\text{ai. } \frac{1}{1073741824}\text{aj. } \frac{1}{2147483648}\text{ak. } \frac{1}{4294967296}\text{al. } \frac{1}{8589934592}\text{am. } \frac{1}{17179869184}\text{an. } \frac{1}{34359738368}\text{ao. } \frac{1}{68719476736}\text{ap. } \frac{1}{137438953472}\text{aq. } \frac{1}{274877906944}\text{ar. } \frac{1}{549755813888}\text{as. } \frac{1}{1099511627776}\text{at. } \frac{1}{2199023255552}\text{au. } \frac{1}{4398046511104}\text{av. } \frac{1}{8796093022208}\text{aw. } \frac{1}{17592186044416}\text{ax. } \frac{1}{35184372088832}\text{ay. } \frac{1}{70368744177664}\text{az. } \frac{1}{140737488355328}\text{ba. } \frac{1}{281474976710656}\text{bb. } \frac{1}{562949953421312}\text{bc. } \frac{1}{1125899906842624}\text{bd. } \frac{1}{2251799813685248}\text{be. } \frac{1}{4503599627370496}\text{bf. } \frac{1}{9007199254740992}\text{bg. } \frac{1}{18014398509481984}\text{bh. } \frac{1}{36028797018963968}\text{bi. } \frac{1}{72057594037927936}\text{bj. } \frac{1}{144115188075855872}\text{bk. } \frac{1}{288230376151711744}\text{bl. } \frac{1}{576460752303423488}\text{bm. } \frac{1}{1152921504606846976}\text{bn. } \frac{1}{2305843009213693952}\text{bo. } \frac{1}{4611686018427387904}\text{bp. } \frac{1}{9223372036854775808}\text{bq. } \frac{1}{18446744073709551616}\text{br. } \frac{1}{36893488147419103232}\text{bs. } \frac{1}{73786976294838206464}\text{bt. } \frac{1}{147573952589676412928}\text{bu. } \frac{1}{295147905179352825856}\text{bv. } \frac{1}{590295810358705651712}\text{bw. } \frac{1}{1180591620717411303424}\text{bx. } \frac{1}{2361183241434822606848}\text{by. } \frac{1}{4722366482869645213696}\text{bz. } \frac{1}{9444732965739290427392}\text{ca. } \frac{1}{18889465931478580854784}\text{cb. } \frac{1}{37778931862957161709568}\text{cc. } \frac{1}{75557863725914323419136}\text{cd. } \frac{1}{151115727451828646838272}\text{ce. } \frac{1}{302231454903657293676544}\text{cf. } \frac{1}{604462909807314587353088}\text{cg. } \frac{1}{1208925819614629174706176}\text{ch. } \frac{1}{2417851639229258349412352}\text{ci. } \frac{1}{4835703278458516698824704}\text{cj. } \frac{1}{9671406556917033397649408}\text{ck. } \frac{1}{19342813113834066795298816}\text{cl. } \frac{1}{38685626227668133590597632}\text{cm. } \frac{1}{77371252455336267181195264}\text{cn. } \frac{1}{154742504910672534362390528}\text{co. } \frac{1}{309485009821345068724781056}\text{cp. } \frac{1}{618970019642690137449562112}\text{cq. } \frac{1}{1237940039285380274899124224}\text{cr. } \frac{1}{2475880078570760549798248448}\text{cs. } \frac{1}{4951760157141521099596496896}\text{ct. } \frac{1}{9903520314283042199192993792}\text{cu. } \frac{1}{19807040628566084398385987584}\text{cv. } \frac{1}{39614081257132168796771975168}\text{cw. } \frac{1}{79228162514264337593543950336}\text{cx. } \frac{1}{158456325028528675187087900672}\text{cy. } \frac{1}{316912650057057350374175801344}\text{cz. } \frac{1}{633825300114114700748351602688}\text{da. } \frac{1}{1267650600228229401496703205376}\text{db. } \frac{1}{2535301200456458802993406410752}\text{dc. } \frac{1}{5070602400912917605986812821504}\text{dd. } \frac{1}{10141204801825835211973625643008}\text{de. } \frac{1}{20282409603651670423947251286016}\text{df. } \frac{1}{40564819207303340847894502572032}\text{dg. } \frac{1}{81129638414606681695789005144064}\text{dh. } \frac{1}{162259276829213363391578010288128}\text{di. } \frac{1}{324518553658426726783156020576256}\text{dj. } \frac{1}{649037107316853453566312041152512}\text{dk. } \frac{1}{1298074214633706907132624082305024}\text{dl. } \frac{1}{2596148429267413814265248164610048}\text{dm. } \frac{1}{5192296858534827628530496329220096}\text{dn. } \frac{1}{10384593717069655257060992658440192}\text{do. } \frac{1}{20769187434139310514121985316880384}\text{dp. } \frac{1}{41538374868278621028243970633760768}\text{dq. } \frac{1}{83076749736557242056487941267521536}\text{dr. } \frac{1}{166153499473114484112975882535043072}\text{ds. } \frac{1}{332306998946228968225951765070086144}\text{dt. } \frac{1}{664613997892457936451903530140172288}\text{du. } \frac{1}{1329227995784915872903807060280344576}\text{dv. } \frac{1}{2658455991569831745807614120560689152}\text{dw. } \frac{1}{5316911983139663491615228241121378304}\text{dx. } \frac{1}{10633823966279326983230456482242756608}\text{dy. } \frac{1}{21267647932558653966460912964485513216}\text{dz. } \frac{1}{42535295865117307932921825928971026432}\text{ea. } \frac{1}{85070591730234615865843651857942052864}\text{eb. } \frac{1}{170141183460469231731687303715884105728}\text{ec. } \frac{1}{340282366920938463463374607431768211456}\text{ed. } \frac{1}{680564733841876926926749214863536422912}\text{ee. } \frac{1}{1361129467683753853853498429727072845824}\text{ef. } \frac{1}{2722258935367507707706996859454145691648}\text{eg. } \frac{1}{5444517870735015415413993718908291383296}\text{eh. } \frac{1}{10889035741470030830827987437816582766592}\text{ei. } \frac{1}{21778071482940061661655974875633165533184}\text{ej. } \frac{1}{43556142965880123323311949751266331066368}\text{ek. } \frac{1}{87112285931760246646623899502532662132736}\text{el. } \frac{1}{174224571863520493293247799005065324265472}\text{em. } \frac{1}{348449143727040986586495598010130648530944}\text{en. } \frac{1}{696898287454081973172991196020261297061888}\text{eo. } \frac{1}{1393796574908163946345982392040522594123776}\text{ep. } \frac{1}{2787593149816327892691964784081045188247552}\text{eq. } \frac{1}{5575186299632655785383929568162090376495104}\text{er. } \frac{1}{11150372599265311570767859136324180752990208}\text{es. } \frac{1}{22300745198530623141535718272648361505980416}\text{et. } \frac{1}{44601490397061246283071436545296723011960832}\text{eu. } \frac{1}{89202980794122492566142873090593446023921664}\text{ev. } \frac{1}{178405961588244985132285746181186892047843328}\text{ew. } \frac{1}{356811923176489970264571492362373784095686656}\text{ex. } \frac{1}{713623846352979940529142984724747568191373312}\text{ey. } \frac{1}{1427247692705959881058285969449495136382746624}\text{ez. } \frac{1}{2854495385411919762116571938898990272765493248}\text{fa. } \frac{1}{5708990770823839524233143877797980545530986496}\text{fb. } \frac{1}{11417981541647679048466287755595961091061972992}\text{fc. } \frac{1}{22835963083295358096932575511191922182123945984}\text{fd. } \frac{1}{45671926166590716193865151022383844364247891968}\text{fe. } \frac{1}{91343852333181432387730302044767688728495783936}\text{ff. } \frac{1}{182687704666362864775460604089535377456991567872}\text{fg. } \frac{1}{365375409332725729550921208179070754913983135744}\text{fh. } \frac{1}{730750818665451459101842416358141509827966271488}\text{fi. } \frac{1}{1461501637330902918203684832716283019655932542976}\text{fj. } \frac{1}{2923003274661805836407369665432566039311865085952}\text{fk. } \frac{1}{5846006549323611672814739330865132078623730171904}\text{fl. } \frac{1}{11692013098647223345629478661730264157247460343808}\text{fm. } \frac{1}{23384026197294446691258957323460528314494920687616}\text{fn. } \frac{1}{46768052394588893382517914646921056628989841375232}\text{fo. } \frac{1}{93536104789177786765035829293842113257979682750464}\text{fp. } \frac{1}{187072209578355573530071658587684226515959365500928}\text{fq. } \frac{1}{374144419156711147060143317175368453031918731001856}\text{fr. } \frac{1}{748288838313422294120286634350736906063837462003712}\text{fs. } \frac{1}{1496577676626844588240573268701473812127674924007424}\text{ft. } \frac{1}{2993155353253689176481146537402947624255349848014848}\text{fu. } \frac{1}{5986310706507378352962293074805895248510699696029696}\text{fv. } \frac{1}{11972621413014756705924586149611790497021399392059392}\text{fw. } \frac{1}{23945242826029513411849172299223580994042798784118784}\text{fx. } \frac{1}{47890485652059026823698344598447161988085597568237568}\text{fy. } \frac{1}{95780971304118053647396689196894323976171195136475136}\text{fz. } \frac{1}{191561942608236107294793378393788647952342390272950272}\text{ga. } \frac{1}{383123885216472214589586756787577295904684780545900544}\text{gb. } \frac{1}{766247770432944429179173513575154591809369561091801088}\text{gc. } \frac{1}{1532495540865888858358347027150309183618739122183602176}\text{gd. } \frac{1}{3064991081731777716716694054300618367237478244367204352}\text{ge. } \frac{1}{6129982163463555433433388108601236734474956488734408704}\text{gf. } \frac{1}{12259964326927110866866776217202473468949912977468817408}\text{gg. } \frac{1}{24519928653854221733733552434404946937899825954937634816}\text{gh. } \frac{1}{49039857307708443467467104868809893875799651909875269632}\text{gi. } \frac{1}{98079714615416886934934209737619787751599303819750539264}\text{gj. } \frac{1}{196159429230833773869868419475239575503198607639501078528}\text{gk. } \frac{1}{392318858461667547739736838950479151006397215279002157056}\text{gl. } \frac{1}{784637716923335095479473677900958302012794430558004314112}\text{gm. } \frac{1}{1569275433846670190958947355801916604025588861116008628224}\text{gn. } \frac{1}{3138550867693340381917894711603833208051177722232017256448}\text{go. } \frac{1}{6277101735386680763835789423207666416102355444464034512896}\text{gp. } \frac{1}{12554203470773361527671578846415332832204710888928069025792}\text{gq. } \frac{1}{25108406941546723055343157692830665664409421777856138051584}\text{gr. } \frac{1}{50216813883093446110686315385661331328818843555712276103168}\text{gs. } \frac{1}{100433627766186892221372630771322662657637687111424552206336}\text{gt. } \frac{1}{200867255532373784442745261542645325315275374222849104412672}\text{gu. } \frac{1}{401734511064747568885490523085290650630550748445698208825344}\text{gv. } \frac{1}{803469022129495137770981046170581301261101496891396417650688}\text{gw. } \frac{1}{1606938044258990275541962092341162602522202993782792835301376}\text{gx. } \frac{1}{3213876088517980551083924184682325205044405987565585670602752}\text{gy. } \frac{1}{6427752177035961102167848369364650410088811975131171341205504}\text{gz. } \frac{1}{12855504354071922204335696738729300820177623950262342682411008}\text{ha. } \frac{1}{25711008708143844408671393477458601640355247900524685364822016}\text{hb. } \frac{1}{51422017416287688817342786954917203280710495801049370729644032}\text{hc. } \frac{1}{102844034832575377634685573909834406561420991602098741459288064}\text{hd. } \frac{1}{205688069665150755269371147819668813122841983204197482918576128}\text{he. } \frac{1}{411376139330301510538742295639337626245683966408394965837152256}\text{hf. } \frac{1}{822752278660603021077484591278675252491367932816789931674304512}\text{hg. } \frac{1}{1645504557321206042154969182557350504982735865633579863348609024}\text{hh. } \frac{1}{3291009114642412084309938365114701009965471731267159726697218048}\text{hi. } \frac{1}{6582018229284824168619876730229402019930943462534319453394436096}\text{hj. } \frac{1}{13164036458569648337239753460458804039861886925068638906788872192}\text{hk. } \frac{1}{26328072917139296674479506920917608079723773850137277813577744384}\text{hl. } \frac{1}{52656145834278593348959013841835216159447547700274555627155488768}\text{hm. } \frac{1}{105312291668557186697918027683670432318895095400549111254310977536}\text{hn. } \frac{1}{210624583337114373395836055367340864637790190801098222508621955072}\text{ho. } \frac{1}{421249166674228746791672110734681729275580381602196445017243910144}\text{hp. } \frac{1}{842498333348457493583344221469363458551160763204392890034487820288}\text{hq. } \frac{1}{1684996666696914987166688442938726917102321526408785780068975640576}\text{hr. } \frac{1}{3369993333393829974333376885877453834204643052817571560137951281152}\text{hs. } \frac{1}{6739986666787659948666753771754907668409286105635143120275902562304}\text{ht. } \frac{1}{13479973333575319897333507543509815336818572211270286240551805124608}\text{hu. } \frac{1}{26959946667150639794667015087019630673637144422540572481103610249216}\text{hv. } \frac{1}{53919893334301279589334030174039261347274288845081144962207220498432}\text{hw. } \frac{1}{107839786668602559178668060348078522694548577690162289924414440996864}\text{hx. } \frac{1}{215679573337205118357336120696157045389097155380324579848828881993728}\text{hy. } \frac{1}{431359146674410236714672241392314090778194310760649159697657763987456}\text{hz. } \frac{1}{862718293348820473429344482784628181556388621521298319395315527974912}\text{ia. } \frac{1}{172543658669764094685868896556925636311$

2. If $3\frac{1}{2}$ yards of cloth, that is $1\frac{3}{4}$ yards broad, will make a cloak; how many yards of shalloon, that is $\frac{1}{4}$ yards wide, will line it?

$$\begin{array}{ccc} \text{yds.} & \text{yds.} & \text{yds.} \\ \text{As } \frac{1}{4} & : & 1\frac{3}{4} :: 3\frac{1}{2} \\ \text{Here } 1\frac{3}{4} = \frac{7}{4}, \text{ and } 3\frac{1}{2} = \frac{7}{2}. \end{array}$$

$$\text{Then } \frac{4 \times 7 \times 7}{5 \times 4 \times 2} = \frac{49}{10} = 4 \text{ yds. } 3\frac{3}{5} \text{ qrs. } \text{Ans.}$$

3. Bought $\frac{3}{4}$ yards for 12s. 6d.; what is the value of $2\frac{1}{4}$ yards? *Ans. £1 19s. 7d.*

4. What is the value of $\frac{3}{8}$ of a yard of cloth, if $2\frac{1}{4}$ yards cost £1 $\frac{1}{4}$? *Ans. 5s. 1 $\frac{1}{4}$ d. $\frac{5}{11}$*

5. Bought $5\frac{1}{2}$ yards for £4 15s., what is the value of $3\frac{3}{4}$ yards? *Ans. £2 18s. 7 $\frac{1}{4}$ d. $\frac{29}{11}$*

6. Bought $\frac{3}{4}$ of a yard for $\frac{7}{8}$ of a £. what is the value of $\frac{3}{4}$ of an English ell? *Ans. £1 1s. 10 $\frac{1}{2}$ d.*

7. Bought $\frac{3}{5}$ of a Flemish ell for 7s. 6d.; what will $2\frac{1}{2}$ yds. cost, at the same rate? *Ans. £2 6s. 8d.*

8. How much in length, that is $\frac{3}{4}$ yards wide, is equal to $20\frac{1}{2}$ yards in length and $\frac{7}{8}$ yards wide? *Ans. 23 yds. 3 $\frac{1}{2}$ qrs.*

9. Bought 7 yards, 3 qrs. of cotton to line a garment, which contained 9 yards 2 qrs. of stuff $\frac{3}{4}$ wide; what was the width of the cotton? *Ans. 3 qrs. 2 $\frac{3}{4}$ n.*

10. A floor measures 18 feet 6 inches in length and 15 feet 9 inches in width; how many yards of carpet, which measures $2\frac{1}{2}$ feet wide, will cover it? *Ans. 35 yds. 11 $\frac{5}{11}$ in.*

11. What is the value of $3\frac{1}{2}$ yards of cloth, if I give £4 6s. 8d. for $9\frac{1}{4}$ yards? *Ans. £1 10s.*

12. Bought 3 cwt. 3 qrs. 14 lb. of sugar for £10 13s. 4d.; what is the value of 3 qrs. 14 lb. of the same? *Ans. £2 8s. 2 $\frac{3}{4}$ d.*

13. Bought $\frac{2}{3}$ of a mine, and sold $\frac{2}{3}$ of my share for £520; required the value of the whole mine. *Ans. £1300*

14. Bought $\frac{3}{8}$ of a mill for £2400; what is the value of the remainder? *Ans. £4000*

15. Bought $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ of an English ell of cloth for 12s. 6d.; what is the value of $7\frac{1}{2}$ yds. of the same? *Ans. £11 16s. 3d.*

16. Borrowed of my friend £36 2s. 6d. for 9 months; how long must I lend him £25 17s. 6d., to requite his kindness? *Ans. 12 $\frac{1}{2}$ months*

17. Borrowed of my friend £25 12s. 6d. for 8 months; what length of time shall I lend him £100 to requite his kindness? *Ans.* $2\frac{1}{2}$ months

18. Bought $7\frac{1}{4}$ yards of cloth for £7 2s. 6d.; what will 4 pieces cost, each measuring $24\frac{3}{4}$ yards, at the same rate? *Ans.* £88 4s. $3\frac{1}{4}$ d.

19. A gentleman gave his son John $\frac{1}{3}$ of his estate; to his son Thomas $\frac{1}{4}$ of the remainder; and the remainder, which was £2000, to his daughter; required each son's share, and the value of the estate.

Ans. £15000 John's, £8000 Thomas', and £25000 whole.

20. A gentleman gave $\frac{1}{3}$ of his estate to his son, $\frac{1}{4}$ of the remainder to his daughter, and the residue in small legacies; when these legacies came to be paid, the son received £13000 more than the daughter. What was each child's share, the sum left in small legacies, and value of the testator's property? — *Ans.* £25000 son's, £12000 daughter's, £3000 small legacies, and £40000 whole property.

DOUBLE RULE OF THREE IN VULGAR FRACTIONS.

Rule as per whole numbers.

EXAMPLES.

1. Received £1 17s. 6d. for the interest of £75 for six months; what will be the interest of £125 for nine months?

$$\begin{array}{rcl} \text{£.} & \text{month.} & \text{£. s. d.} \\ \text{As } 75 & : 6 & :: 1 \quad 17 \quad 6 \\ 125 & : 9 & :: \hline \end{array}$$

$$\text{Here } £1 \text{ 17s. 6d.} = 1\frac{17}{20} = £1\frac{17}{20}$$

$$\text{Then } \frac{1 \times 1 \times 17 \times 17 \times 9}{17 \times 6 \times 8 \times 1 \times 1} = £\frac{225}{48} = £4 \text{ 13s. 9d.}$$

2. Paid £2 13s. 4d. for the carriage of 25 cwt. 2 qrs. 14 lbs. $120\frac{1}{4}$ miles,—how much can I have carried $362\frac{1}{4}$ miles for £3 6s. 8d.?

$$\begin{array}{rcl} \text{cwt.} & \text{qrs.} & \text{lbs.} & \text{miles.} & \text{£. s. d.} \\ \text{As } 25 & 2 & 14 & : 120\frac{1}{4} & :: 2 \quad 13 \quad 4 \\ \hline & & & : 362\frac{1}{4} & :: 3 \quad 6 \quad 8 \end{array}$$

Here 25 cwt. 2 qrs. 14 lbs. = $25\frac{1}{2} = 2\frac{5}{8}$ cwt., $120\frac{1}{2} = 4\frac{1}{2}$ miles, £2 13s. 4d. = $2\frac{1}{3} = \frac{2}{3}$, $362\frac{1}{4} = 1\frac{1}{4}$ miles, and £3 6s. 8d. = $3\frac{1}{3} = £1\frac{1}{3}$.

Then $\frac{3 \times 4 \times 205 \times 483 \times 10}{8 \times 1449 \times 8 \times 4 \times 3} = \frac{2050}{192} = 10$ cwt.
2 qr. $19\frac{1}{2}$ lb.

3. If 13 men in $7\frac{1}{2}$ days, earn £3 10s.—how much will 10 men earn in $30\frac{1}{2}$ days?
Ans. £11 0s. $2\frac{6}{7}$

NOTE.—For practice, the student may solve, by fractions, the questions given in the simple rules in Compound Proportion.

DECIMAL FRACTIONS. .

A DECIMAL FRACTION has for its denominator a unit, with as many ciphers annexed to the right of it, as the numerator contains figures; and is expressed by writing the numerator only, with a point placed before it; as $\frac{1}{10} = .1$; $\frac{5}{10} = .5$; $\frac{75}{100} = .75$, &c.

In Decimal fractions, unity is divided into ten equal parts, called tenths, and these tenths are each subdivided into ten other equal parts, called hundredths; each of these hundredths into ten equal parts called thousandths, &c. decreasing in value, from left to right, in a tenfold proportion.

Tens	Units.		Tens.	Hundreds.	Thousands.	Tens of thousands.	Hundreds of thousands.	Millions.	Tens of millions.	Hundreds of millions.	&c. &c.
2	3	.	4	5	6	7	8	9	1	5	

Here 23 is a whole number and .4 5 6 7 8 9 1 5 a decimal, with the point or period placed before it.

The above table may be decomposed as follows :

The	2	denotes	20	
—	3	ditto	3	
—	4	ditto	.4	$= \frac{4}{10}$
—	5	ditto	.05	$= \frac{5}{100}$
—	6	ditto	.006	$= \frac{6}{1000}$
—	7	ditto	.0007	$= \frac{7}{10000}$
—	8	ditto	.00008	$= \frac{8}{100000}$
—	9	ditto	.000009	$= \frac{9}{1000000}$
—	1	ditto	.0000001	$= \frac{1}{10000000}$
And	5	ditto	.00000005	$= \frac{5}{100000000}$
				$\frac{45678915}{100000000}$
23.45678915				$= 23 \frac{45678915}{100000000}$

NOTE.—Ciphers placed to the right hand of a decimal neither increase nor decrease its value, as .5 denotes five tenth's or $\frac{5}{10} = \frac{1}{2}$

$$.50 = \frac{50}{100} = \frac{1}{2}$$

$$.500 = \frac{500}{1000} = \frac{1}{2}$$

But ciphers placed to the left hand of a decimal, decrease its value.

$$\text{Thus } .5 = \frac{5}{10} = \frac{1}{2}$$

$$.05 = \frac{5}{100} = \frac{1}{20}$$

$$.005 = \frac{5}{1000} = \frac{1}{200}$$

Decimals are divided into 3 sorts, viz. *Finite, Infinite, and Repetends.*

1. *A Finite Decimal* is one which terminates, as .5 ; .25 ; .75 ; &c.

2. *An Infinite Decimal* is one which does not terminate, and has plus generally placed at the end of it, as $.78539 +$

3. *A repeating Decimal, or repetend*, is when one, two, or more figures continually repeat, as $.3333$, &c., which is called a single repetend, and is generally written thus, $.3$

Also, $.454545$ &c. $= .4\bar{5}$

and $.267267$ &c. $= .26\bar{7}$; and so on, which are compound repetends.

Again, $.41333$ &c. $= .41\bar{3}$, and 4.21666 &c. $= 4.21\bar{6}$, are compound or mixed repetends, having significant figures between the repetend and decimal point, or a whole number before the repetend.

ADDITION OF DECIMALS.

RULE.—Place the figures in such order, that those of the same denominations may stand under each other; add them together as in whole numbers, and place the decimal point in the sum under the other points.

EXAMPLES.

1. Add 3.45 ; 46.789 ; $.4689$; 468.7 and $.25$ together.

$$\begin{array}{r} 3.45 \\ 46.789 \\ .4689 \\ 468.7 \\ .25 \\ \hline \end{array}$$

519.6579

2. Add 4.37 ; 56.785 ; $.3724$; 865.72 ; 467.846 ; and $.72$ together. *Ans.* 1395.8134.

3. Add $.34$; $.867$; $.4956$; $.78546$; 2.68 ; and 378.42 together. *Ans.* 383.58806

4. Add $.37$; $.4689$; $.5$; $.864$; 23 ; 785 ; and 745.8 together. *Ans.* 1556.0029.

5. Add 3.67 ; $.056$; 46.7234 ; 634.8 ; and $.78$ together. *Ans.* 686.0294.

6. Add $.4$; $.05$; $.006$; $.0007$; $.00008$; and $.000009$ together. *Ans.* .456789.

SUBTRACTION OF DECIMALS.

RULE.—Place the numbers in such order, that the decimal point in the greater number may be over the decimal point in the less, and proceed as in Simple Subtraction, placing the point in the difference under the other points.

EXAMPLES.

1.

From	46.728
Take	2.3467
44,3813	
2. Subtract 4.728 from 29.4678. *Ans.* 24.7398.
3. Required the difference between 42,849 and .956. *Ans.* 41.893.
4. Required the difference between 14.469 and 36.7854. *Ans.* 22.3164.
5. Required the sum and difference of 24.698 and 39.12345. *Ans.* 63.82145 sum, 14.42545 difference.
6. What number added to .7854 will make 2? *Ans.* 1.2146.

MULTIPLICATION OF DECIMALS.

RULE.—Place the given numbers as in Simple Multiplication, and find their product.

Point off as many figures, from right to left, in the product, for decimal places, as there are decimals in both the multiplier and multiplicand.

If there be not as many figures in the product as there are decimal places in the multiplier and multiplicand, make up the deficiency by annexing ciphers to the left of the product, as in Example 2nd.

EXAMPLES.

- | | |
|---|---|
| <p>1. Multiply 4.2683</p> <p style="padding-left: 100px;">by 4.25</p> <hr style="width: 100px; margin: 0;"/> <p style="padding-left: 100px;">213415</p> <p style="padding-left: 100px;">85366</p> <p style="padding-left: 100px;">170732</p> <hr style="width: 100px; margin: 0;"/> <p style="padding-left: 100px;">18.140275</p> | <p>2. Multiply .3426</p> <p style="padding-left: 100px;">by .0045</p> <hr style="width: 100px; margin: 0;"/> <p style="padding-left: 100px;">17130</p> <p style="padding-left: 100px;">13704</p> <hr style="width: 100px; margin: 0;"/> <p style="padding-left: 100px;">.00154170</p> |
|---|---|

3. Multiply 46.784 by 3.65 *Ans.* 170.7616.
4. Multiply 7.3284 by .049 *Ans.* .3590916
5. Multiply 75.6842 by 3.678 *Ans.* 278.3664876.
6. Multiply .84623 by 42.5 *Ans.* 35.964775.
7. Multiply .004 by 0004 *Ans.* .0000016.
8. Multiply 3.467 by 4.69 *Ans.* 16.26023.
9. Required the sum, difference, and product, of 4.56 and 3.5
Ans. 8.06 sum, 1.06 diff. 15.96 prod.
10. Find the sum, difference, and product, of .06 and .428571
Ans. .488571 sum, .368571 diff. and .02571426 prod.

DIVISION OF DECIMALS.

RULE.—1. Divide as in whole numbers, and point off as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor. See Example 1st.

2. If the quotient have not so many places of figures in it, as there are decimal places in the dividend more than the divisor, add ciphers to the left of the quotient, to make up the deficiency, as in Example 2nd.

3 If the divisor contain more figures than the dividend, add as many ciphers to it as may be required in order to perform the division, as in Example 3rd.

4. If there be not as many decimal places in the dividend as in the divisor, make the decimal places equal by annexing ciphers, as in Example 4th.

5 If there be a remainder after division, ciphers may be added to the right of it, and proceed with the division. These ciphers are decimals. If the dividend be a repetend, the repeating figures must be added instead of ciphers, as in Example 5th.

EXAMPLES.

1. Divide 43.682 by 2.35.

$$2.35 \overline{) 43.682} (18.5$$

235

2018

1880

1382

1175

207

2. Divide .23456 by 43.6

$$43.6 \overline{) .23456} (.0053$$

2180

1656

1308

248

3. Divide 46.2 by 6.268

$$\begin{array}{r} 6.268 \overline{) 46.200} \quad (7.37077 \\ 43876 \end{array}$$

$$\begin{array}{r} 23240 \\ 18804 \end{array}$$

$$\begin{array}{r} 44360 \\ 43876 \end{array}$$

$$\begin{array}{r} 48400 \\ 43876 \end{array}$$

$$\begin{array}{r} 45240 \\ 43876 \end{array}$$

$$\begin{array}{r} 1364 \end{array}$$

4. Divide 125 by .25

$$\begin{array}{r} .25 \overline{) 125.00} \quad (500 \\ 125 \end{array}$$

$$\begin{array}{r} 00 \end{array}$$

5. Divide 13.46 by 6

$$\begin{array}{r} 6 \overline{) 13.46} \\ 2.2441077 \end{array}$$

6. Divide 42.468 by .4

$$\text{Ans. } 106.17$$

7. Divide .78464 by .5

$$\text{Ans. } 1.56928$$

8. Divide 2.3467 by 8

$$\text{Ans. } .2933375$$

9. Divide .0469 by 1.2

$$\text{Ans. } .03908\dot{3}$$

10. Divide .1468 by 1.5

$$\text{Ans. } .09786$$

11. Divide 7.468 by .25

$$\text{Ans. } 29.872$$

12. Divide .0008136 by 678

$$\text{Ans. } .0000012$$

13. Divide 7854 by 10000

$$\text{Ans. } .7854$$

14. Divide 14.4 by 6.84

$$\text{Ans. } 2.11176 +$$

15. Express the quotient of 1694 .647 by 46.849.

$$\text{Ans. } 36.172546$$

16. What is the sum, difference, product, and quotient, of .0469 and 1.2? *Ans.* 1.2469 sum, 1.1531 difference, .05628 product, .039083 or 25.58 + quotient.

REDUCTION OF DECIMALS.

CASE I.—To reduce a given vulgar fraction to a decimal, of the same value.

RULE.—Divide the numerator, with as many ciphers annexed to the right of it for decimals, as may be required to perform the division; and point of the decimal places in the quotient as in division.

NOTE.—When the quotient becomes a repetend, if a single one, place a dot over the repeated figure; if a compound one, place dots over the first and last figures of the repetend, as in Example 2nd.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ to decimals of the same value.

$$\begin{array}{r} 4 \overline{)1.00} \\ \underline{} \\ .25 \end{array}$$

$$\begin{array}{r} 2 \overline{)1.0} \\ \underline{} \\ .5 \end{array}$$

$$\begin{array}{r} 4 \overline{)3.00} \\ \underline{} \\ .75 \end{array}$$

2. Reduce $\frac{2}{3}$ and $\frac{4}{7}$ to decimals of the same value.

$$\begin{array}{r} 3 \overline{)2.0} \\ \underline{} \\ .6 \end{array}$$

$$\begin{array}{r} 7 \overline{)4.000000} \\ \underline{} \\ .571428 \end{array}$$

3. Reduce $\frac{3}{8}$ to a decimal of the same value.

Ans. .375

4. Reduce $\frac{7}{15}$ to a decimal of the same value.

Ans. .466

5. Reduce $\frac{9}{25}$ to a decimal of the same value.

Ans. .36

6. Reduce $\frac{3}{4}$ of $\frac{3}{4}$ to a decimal of the same value.

Ans. .5

7. Reduce $\frac{4}{15}$ to a decimal.

Ans. .032

8. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ to a decimal.

Ans. .25

9. Reduce $\frac{2\frac{1}{2}}{3\frac{3}{4}}$ to a decimal.

Ans. .681

10. Reduce $7\frac{3}{8}$ to a decimal.

Ans. 7.375

11. Reduce $\frac{2}{3}$ of $\frac{1\frac{1}{2}}{5}$ of $1\frac{1}{2}$ to a decimal.

Ans. 3

12. Reduce $\frac{3\frac{1}{4}}{4}$ of $\frac{2}{7}$ of $\frac{1\frac{1}{2}}{2\frac{1}{4}}$ to a decimal.

Ans. .160714285.

13. What is the sum, difference, product, and quotient of $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{3}{4}$ in decimals?

Ans. 1.1 sum, .1 diff. .3 prod. 1.2 quotient.

14. What is the sum, difference, product, and quotient of $\frac{4}{5}$ of $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{3}{8}$ in decimals?

Ans. .7 sum, .1 diff. .148 prod. 1.3 quotient.

15. Express in a decimal $2 + \frac{4}{5} + \frac{6}{1000} + \frac{16}{200000}$.

Ans. 2.8068.

CASE 2.—*To reduce a decimal to its equivalent vulgar fraction.*

RULE.—1. If the given decimal be finite, place 1 with as many ciphers annexed to it as the decimal contains places, for a denominator, which will form the fraction required; as in Example 1st.

2. If the given decimal be a pure repetend, place as many nines under it, for a denominator, as the decimal contains figures; as in Example 2nd.

3. If the given decimal be a mixed repetend, subtract the finite figures from the given number, as in Simple Subtraction, and the remainder will be the numerator of the required fraction. Place as many nines as there are repetends, annexing as many ciphers to the right of them as there are finite decimals, for a denominator, as in Examples 3rd and 4th.

NOTE.—Reduce .234545, &c., add infinitum, to its equivalent vulgar fraction

$$\begin{aligned} \text{Let } s &= .234545, \&c. \&c. \\ \text{Then } 10000s &= 2345.4545, \&c. \\ \text{And } 100s &= 23.4545, \&c. \\ \text{By subtraction, } 9900s &= 2322; \\ &\therefore s = \frac{2322}{9900} \end{aligned}$$

EXAMPLES.

1. Reduce .5 and .05 to their equivalent vulgar fractions.

$$\text{Here } .5 = \frac{5}{10} = \frac{1}{2}, \text{ and } .05 = \frac{5}{100} = \frac{1}{20}.$$

2. Reduce .3 and .285714 to equivalent vulgar fractions.

$$\text{Hence } .3 = \frac{3}{10} = \frac{3}{10}, \text{ and } .285714 = \frac{285714}{999999} = \frac{2}{7}.$$

3. Reduce .26 to its equivalent vulgar fraction.

$$\text{Hence } .2\dot{6} = \frac{26-2}{90} = \frac{24}{90} = \frac{4}{15}.$$

4. Reduce 5.26 to its equivalent vulgar fraction.

$$\text{Here } 5.26 = \frac{526-52}{90} = \frac{474}{90} = \frac{79}{15}$$

5. Reduce .25 to its equivalent vulgar fraction. *Ans.* $\frac{1}{4}$

6. Reduce .75 to its equivalent vulgar fraction. *Ans.* $\frac{3}{4}$

7. Reduce .375 to its equivalent vulgar fraction. *Ans.* $\frac{3}{8}$

8. Reduce .875 to its equivalent vulgar fraction. *Ans.* $\frac{7}{8}$

9. Reduce .6 to its equivalent vulgar fraction. *Ans.* $\frac{3}{5}$

10. Reduce .28 to its equivalent vulgar fraction. *Ans.* $\frac{28}{100} = \frac{7}{25}$

11. Reduce 416 to its equivalent vulgar fraction. *Ans.* $\frac{5}{7}$

12. Reduce 3.125 to its equivalent vulgar fraction. *Ans.* $2\frac{1}{8}$

NOTE.—This case enables us to add, subtract, multiply, and divide repetends with accuracy.

13. Required the sum, difference, product, and quotient of .285714 and .2142857.

To find the sum.

$$\begin{array}{rcl} \text{Here } .285714 & = & \frac{2}{7} \\ \text{and } .2142857 & = & \frac{3}{14} \end{array} \quad \begin{array}{l} \text{or,} \\ \text{or,} \end{array} \quad \begin{array}{r} .2857142 \\ .2142857 \end{array}$$

$$\text{Then } \frac{2}{7} + \frac{3}{14} = \frac{7}{14} = .5 \text{ sum.}$$

$$\begin{array}{r} .4999999 \text{ sum.} \end{array}$$

To find the difference.

$$\text{Here } \frac{2}{7} - \frac{3}{14} = \frac{1}{14} = .0714285 \text{ diff.} \quad \begin{array}{r} .2857142 \\ .2142857 \\ \hline .0714285 \end{array}$$

To find the product.

$$\text{Here } \frac{2}{7} \times \frac{3}{14} = \frac{3}{49} = .061224489795918367346938775510204081632653 \text{ product.}$$

To find the quotient.

$$\frac{2}{7} \times \frac{14}{3} = \frac{28}{3} = 9\frac{1}{3} = 1.3 \text{ quotient.}$$

CASE 3.—To reduce money, weights, measures, &c. into decimals of a given denomination, retaining their value.

RULE.—Take the lowest term in the given sum, weight, measure, &c.. to which prefix ciphers, and divide by the next superior denomination; to this quotient add the next superior given sum, weight, &c., as a whole number, and divide by the next superior denomination, and so on. See 1st part of the Ex. 1st.

Or, reduce the given sum, weight, measure, &c., into the lowest term mentioned, for a dividend, to which add ciphers, if required; and the given denomination into the same terms for a divisor, and the quotient will be the answer. as in the 2nd. solutions to examples 1st. and 2nd, and the quotient will be the decimal required.

EXAMPLES.

1. Reduce 8s. 7½d. to the decimal of a £.

$$2)1.0$$

$$\text{Or thus, } 8s. 7\frac{1}{2}d.$$

$$20s.$$

$$12$$

$$12$$

$$12)7.5 \text{ Pence.}$$

$$103$$

$$240$$

$$20)8.625 \text{ Shillings.}$$

$$2$$

$$2$$

$$.43125 \text{ £.}$$

$$207$$

$$480$$

Then $207 \div 483 = .43125\text{£}$. *Ans.*

2. Reduce 2 qrs 3 lb. 8 oz. to the decimal of a cwt.

	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>	<i>lb.</i>
16)8.0	2	3	8	112
	28			2
28)3.5	Or			
	59			224
4)2.125	2			

.53125 cwt. 119 half lbs. 224)119.0 (.53125 cwt.

3. Express £3. 5s. 6½d. in decimals. *Ans.* £3.277083
4. Reduce 3 yds. 2 qrs. 2 nails, to the decimal of an English ell. *Ans.* 2.9 English ells.
5. Reduce 6d. to the decimal of a shilling. *Ans.* .5s.
6. Reduce 9½d. to the decimal of a shilling. *Ans.* .7916s.
7. What decimal fraction of a £. is 3s. 6d.? *Ans.* £.175
8. Reduce 17s. 6d. to the decimal of a £. *Ans.* £.875
9. Reduce £2 17s. 4d. to the decimal of a £. *Ans.* £2.86
10. What decimal fraction of a cwt. is 2 qrs. 14 lbs.? *Ans.* .625 cwt.
11. Reduce 3 qrs. 2 nails to the decimal of a yard. *Ans.* .875 yds.
12. Reduce 6 oz. 5 dwt. 4 grs. to the decimal of a lb. *Ans.* .521527 lb.
13. Reduce 3 roods, 15 p. to the decimal of an acre. *Ans.* .84375 a.
14. Reduce 7s. 6½d. to the decimal of a moidore. *Ans.* .27839506172 moidores.
15. Reduce 2s. 6½d. to the decimal of half a guinea, or express 2s. 6½d. decimally, in half guineas. *Ans.* .244047619
16. Reduce $\frac{1}{6}$ of $\frac{6}{7}$ of 14s. 10½d. to the decimal of a £. *Ans.* .10625
17. Reduce 3.145 s. to the fraction of a £. *Ans.* $\frac{1173}{1168}$.
18. Express 41.785 d. in the fraction of a £. *Ans.* $\frac{1571}{1760}$.
19. What decimal of a lb. troy, is 9oz. 12drs. avoirdupois? *Ans.* .7405598 lb. troy.

CASE IV.—*To find the value of a given decimal fraction, in known parts of an integer.*

RULE.—Multiply the given decimal by the number of parts contained in the next inferior denomination; and from the right hand of the product, point off as many figures as there are decimal places in the given decimal. Multiply the decimals thus pointed off, by the next inferior denomination, and from this product cut off the decimal places as before, and so on, till you have brought out the least known parts of the integer; and the whole numbers thus found, will be the value of the decimal required.

NOTE.—If any portion of the given decimal be a repetend, reduce this portion to its equivalent vulgar fraction, by Case 2nd, before you multiply, as in Example 2nd.

EXAMPLES.

1. What is the value of .878125 of a £.?

$$\begin{array}{r}
 \text{£.} \\
 .878125 \\
 20 \\
 \hline
 17.562500 \\
 12 \\
 \hline
 6.750000 \\
 .4 \\
 \hline
 3.000000
 \end{array}$$

Ans. 17s. 6½d.

2. What is the value of .002083 of a lb. troy?

Here by Case 2nd, .002083 = .00208½

$$\begin{array}{r}
 12 \\
 \hline
 .02500 \\
 20 \\
 \hline
 .50000 \\
 24 \\
 \hline
 12.00000
 \end{array}$$

Ans. 12 grs.

3. What is the value of .175 of a £.?

Ans. 3s. 6d.

4. What is the value of .875 of a £.?

Ans. 17s. 6d.

5. What is the value of 2.86 of a £.?

Ans. £2 17s. 4d.

6. What is the value of .625 of a cwt.? *Ans.* 2 qrs. 14 lb.
 7. What is the value of .05875 of a lb. avoird.? *Ans.* 15.04 drs.
 8. What is the value of .02575 of a lb. troy? *Ans.* 6 dwts. 4,32 gr.
 9. What is the value of .04535 of a mile? *Ans.* 14 p. 2 yds. 2 ft. 5 in. 1.128 b. c.
 10. What is the value of .14676 of a lb. troy? *Ans.* 1 oz. 15 dwt. 5.376 gr.
 11. What is the value of .36945 of a cwt.? *Ans.* 1 qr. 13.37890 lb.
 12. What is the sum of ,875 of a £. .175 of a shilling, and .75 of a penny?

$$\begin{array}{r}
 .875 \\
 20 \\
 \hline
 17.500 \\
 .175 \\
 \hline
 17.675 \text{ Shillings} \\
 12 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8.100 \\
 .75 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8.85 \text{ Pence} \\
 4 \\
 \hline
 \end{array}
 \quad \text{Ans. } .17\text{s. } 8\frac{3}{4}\text{d. } .4$$

3.40 Farthings

13. What is the sum of .3678 of a £. .467 of a shilling, .25 of a penny, and .7869 of a farthing? *Ans.* 7s. 10½d.
 14. What is the sum of .474 of a week, .36 of a day, and .7854 of an hour? *Ans.* 3 days, 17 hrs. 3 m. 26 seconds
 15. What is the sum of .4573 of a cwt., .236 of a qr., .75 lb. and .5 of an oz.? *Ans.* 2 qr. 2 lb. 9 oz. 11 dr.
 16. What is the difference of .378 £. and .5678 shillings? *Ans.* 6s. 11½d.
 17. Required the value of .389 £. *Ans.* 7s. 9½d. ⅓
 18. Find the value or limit of .142857 cwt. *Ans.* 16 lb.
 19. Find the value or limit of .49£. .76s. and .87d. *Ans.* 10s. 10½d.

20. What sum added to the sum of £.7854 .85s. and .76842d will make £2? *Ans.* £1. 3s. 4.53558d.

21. Find the value of 0.05 of £3 10s., and express the result as the decimal of a £. *Ans.* 3s. 6d. and .175£.

22. Reduce the following expression to a decimal, having seven places.

$$2 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \&c. \quad \text{Ans. } 2.7182816.$$

RULE OF THREE IN DECIMALS.

RULE.—Reduce the given quantities into decimals, and proceed as in whole numbers. When repetends occur, reduce them to their equivalent vulgar fractions, as in Example 2nd.

EXAMPLES.

1. Bought 1 yd. 3 qr. of cloth for £1 2s. 6d.; what is the value of 17 yds. 2 qrs.?

$$\begin{array}{r} 4)3.0 \\ \hline 1.75 \end{array}$$

$$\begin{array}{r} 12)6.0 \\ \hline 20)2.5 \end{array}$$

$$\begin{array}{r} 4)2.0 \\ \hline 17.5 \end{array}$$

$$\begin{array}{ccccccc} & & & 1,125 & & & \\ \text{yds.} & \text{yds.} & & \text{£.} & \text{£.} & \text{£.} & \text{s.} \\ \text{As } 1.75 & : & 17.5 & :: & 1.125 & : & 11.25 = 11 \quad 5 \text{ Ans.} \end{array}$$

2. Bought sugar at £2 17s. 6½d. per cwt.; what is the value of 5 cwt. 2 qrs. 7 lb.?

$$\begin{array}{r} 2)1.0 \\ \hline 12(6.5 \\ \hline 20)17.5416 \end{array}$$

$$\begin{array}{r} 28)7.0 \\ \hline 4)2.25 \\ \hline 5.5625 \text{ cwt.} \end{array}$$

$$\begin{array}{ccccccc} & & & 2.87708\bar{3} & = & 2.87708\frac{1}{3} & \text{£.} \\ \text{cwt.} & \text{cwt.} & & \text{£.} & \text{£.} & \text{s.} & \text{d.} \\ \text{As } 1 & : & 5.5625 & :: & 2.87708\frac{1}{3} & : & 16 \quad 0 \quad 0\frac{1}{2} \quad .625 \text{ Ans.} \end{array}$$

NOTE—For practice under this rule, the student may work, Decimally, the different Examples given in the Rule of Three Direct, Inverse, and the Double Rule of Three.

INTEREST.

INTEREST is money paid for the use of money ; or, it is a sum of money paid by the borrower to the lender, for the use of his money. It is divided into Simple and Compound.

SIMPLE INTEREST.

The Principal is the sum of money lent. The Rate per Cent. or per £100, is the sum agreed upon between the borrower and the lender, which the former is to pay to the latter, for the use of £100, for one year :—Thus 5 per cent. per annum, means, that the borrower is to pay to the lender £5 for having the use of his £100 one year ; $4\frac{1}{2}$ per cent. per annum means that £4 10s. are to be paid for the use of £100 for one year ; 4 per cent. per annum means, that £4 are to be paid for the use of £100 for one year ; $3\frac{1}{2}$ per cent. per annum, that £3 10s. are to be paid, and so on.

When no rate per cent. is agreed upon, 5 per cent. per annum is always understood.

The Amount is the principal and interest added together.

CASE I.—To find the interest and amount of any sum of money, for one year, or any given number of years.

RULE.—As £100 are to the rate per Cent., so is the given sum to the interest for one year ; multiply this interest by the number of years, and the product will be the interest required.

Or, multiply the given sum by the rate per cent., and this product by the number of years ; divide this last product by 100, and the quotient will be the interest required.

NOTE.—Dividing by 100 is only cutting off two figures from the right hand.

Or, if the rate per cent. be any aliquot part of 100, divide the given sum by such aliquot part, and the quotient will be the interest for one year. If the product of the rate per cent. and time be any aliquot part of 100, divide the given sum by such aliquot part, and the quotient, will be the interest required. See the following table :—

TABLE.

For 5 per cent. take $\frac{1}{20}$ of the sum for the interest.

7½	_____	take $\frac{1}{20}$ and $\frac{1}{2}$ of this	_____
10	_____	take $\frac{1}{10}$	_____
12½	_____	take $\frac{1}{8}$	_____
17½	_____	take $\frac{1}{8}$ and $\frac{1}{10}$	_____

per cent. yrs.

For 5 × 4 = 20, take $\frac{1}{20}$ of the sum for the interest.

5	×	3½	=	17½	, take $\frac{1}{8}$ and $\frac{1}{20}$	_____
6	×	5	=	30	, take $\frac{1}{5}$ and $\frac{1}{2}$ of this	_____
4	×	7½	=	30	, take $\frac{1}{5}$ and $\frac{1}{2}$ of this	_____
5½	×	5	=	27½	, take $\frac{1}{4}$ and $\frac{1}{10}$ of this	_____

EXAMPLES.

1. What is the interest of £365 12s. 6d. for one year, at 5 per cent. per annum?

	£.	£.	£.	s.	d.	Or,	£.	s.	d.
As	100	:	5	::	365	12	6		
					5	5	is	$\frac{1}{20}$	365 12 6
									18 5 7½ <i>Ans.</i>

$$\begin{array}{r} 1,00 \overline{) 18,28 \ 2 \ 6} \\ \underline{20} \end{array}$$

$$\begin{array}{r} 1,00 \overline{) 5,62} \\ \underline{12} \end{array}$$

$$\begin{array}{r} 1,00 \overline{) 7,50} \\ \underline{4} \end{array}$$

$$\begin{array}{r} 1,00 \overline{) 2,00} \end{array}$$

£18 5s. 7½d. *Ans.*

2. What is the interest of £254 17s. 6d. for 5 years, at 4 per cent. per annum?

	£.	£.	£.	s.	d.	£.	s.	d.
As	100	:	4	::	254	17	6	: 10 3 10½

£. s. d.

10 3 10½ Interest for 1 year.

50 19 6 Interest required.

Or thus,

£.	s.	d.
254	17	6
		4
1019	10	0
		5
50,97	10	0
	20	
19,50		
	12	
6,00		

Or thus, by Table, $4 \times 5 = 20$.

£.	s.	d.
20 is $\frac{1}{5}$)	254	17 6
	50	19 6

Ans.

3. What is the interest of £48 15s 9d. for 1 year, at 5 per cent. ? *Ans.* £2 8s. 9½d.

4. What is the amount of £325 15s. 5d. for 4 years, at 5 per cent. per annum ? *Ans.* £390 18s. 6d.

5. What is the interest of £349 10s. for 3½ years, at 4 per cent. per annum ? *Ans.* £48 18s. 7d.

6. What is the amount of £320 12s. 6d. for 5 years, at 5½ per cent. per annum ? *Ans.* £408 15s. 11½d.

7. What is the interest of £1429 15s. 3½d. for 4 years, at £4 12s. 6d. or 4½ per cent. per annum ? *Ans.* £264 10s. 1½d.

8. What is the interest of £325 17s. 6d. for 4¾ years, at £4 17s. 6d., or 4¾ per cent. per annum ? *Ans.* £75 9s. 2½d.

9. What is the interest of £126 17s. 6d. for 7¼ years, at 3½ per cent. per annum ? *Ans.* £32 3s. 10½d.

10. What is the interest of 320 moidores, for 5 years, at 5 per cent. per annum ? *Ans.* £108.

11. What is the interest of $\frac{2}{3}$ of $\frac{1}{2}$ of £357 14s. 5½d., $\frac{1}{3}$ of $\frac{1}{2}$ of £1000, and $\frac{1}{3}$ of 160 quarter guineas, for one year, at 5 per cent. per annum ? *Ans.* £25 2s. 4½d.

12. What is the amount of 367 crowns, $\frac{2}{3}$ of $\frac{1}{2}$ of £17, and $\frac{1}{3}$ of $\frac{1}{2}$ of 27 moidores, at 5 per cent. per annum, for 2½ years ? *Ans.* £138 4s. 1½d.

CASE II.—To find the interest for years and months.

RULE.—Find the interest for the years, as in Case 1st, and take parts for the months; add them together, and their sum will be the interest required.

EXAMPLES.

1. What is the interest of £320 12s. 6d. for 3 years and 8 months, at 5 per cent. per annum?

	£.	£.	£.	s.	d.	£.	s.	d.
As 100 :	5	::	320	12	6	:	16	0 7½
m.	£.	s.	d.					
6 is ½	16	0	7½	Interest for 1 year.				
			3					

	48	1	10½	ditto	for 3 years.
2 is ⅓	8	0	3¾	ditto	for 6 months.
	2	13	5¼	ditto	for 2 months.
	58	15	7½	Interest required.	

Or,

	£.	s.	d.	
5 is ⅕	320	12	6	
m.	16	0	7½	Interest for 1 year.
4 is ¼			3	

4 is ⅓	48	1	10½
	5	6	10½
	5	6	10½

58 15 7½ Interest required.

2. What is the interest of £125 12s. 6d. for 2 years and 9 months, at 4½ per cent. per annum? *Ans.* £15 10s. 11d.

3. What is the interest of £148 6s. 8d. for 2 years and 7 months, at 5 per cent. per annum? *Ans.* £19 3s. 2½d.

4. Lent my friend £74 15s. on the 1st of January. How much do I want this day, July the 1st. for principal and interest? *Ans.* £76 12s. 4½d.

5. What is the interest of £374 15s. for 1 year and 10 months, at 4½ per cent. per annum? *Ans.* £32 12s. 8½d.

6. Lent my friend £327 10s. on the 1st of January, 1841, at 5 per cent. per annum. How much do I want for principal and interest on the 1st of May, 1842? *Ans.* £349 6s. 8d.

CASE III.—*To find the interest of any sum for a given number of days.*

RULE 1.—As 365 days are to the interest of the sum for one year, so is the given time, in days, to the interest required.

NOTE.—When the interest is for any part of a year, the time should be taken in days, excluding or not reckoning the day on which the money is lent, but including the day on which it is paid.

(a) What is the interest of £215, for 13 weeks, at 5 per cent. per annum?

$$5 \text{ is } \frac{1}{20}) 215 \text{ £}$$

10 15 Interest for 1 year.

days. £. s. d. days. £. s. d.

As 365 : 10 15 :: 91 : 2 13 7½ Ans.

RULE 2.—1, Multiply the given sum by the number of days, and divide the product by 7300; the quotient will be the interest, at 5 per cent. per annum.

2. Divide this interest by 5, and multiply the quotient by the given rate per cent; the product will be the interest required.

RULE 3.—Multiply the given sum, the number of days, and twice the rate per cent. continually together; and divide the product by 73000; the quotient will be the interest required.

What is the interest of £356, for 84 days, at 5 per cent. 4½ per cent. and 3½ per cent. per annum?

By Rule 2nd,

356

84

7300)29904 (£4 1s. 11d. Interest at 5 per cent.

5)4 1 11

16 4½

3½

16 4½ Interest at 1 per ct.

4½

2 17 4½ at 3½ per cent.

3 13 8½ Interest at 4½ per cent.

(a) NOTE.—The weeks must be reduced into days, for, if stated in weeks, as follows—

As 52 weeks : £10 15s. :: 13 weeks : £2 13s. 9d.
which is too much

By Rule 3rd.

$$\frac{356 \times 84 \times 5 \times 2}{73000} = \frac{299040}{73000} = \begin{matrix} \text{£} & \text{s.} & \text{d.} \\ 4 & 1 & 11 \end{matrix} \text{ at 5 per cent}$$

$$\frac{356 \times 84 \times 4\frac{1}{2} \times 2}{73000} = \frac{269136}{73000} = \begin{matrix} \text{£} & \text{s.} & \text{d.} \\ 3 & 13 & 8\frac{3}{4} \end{matrix} \text{ at } 4\frac{1}{2} \text{ per cent}$$

$$\frac{356 \times 84 \times 3\frac{1}{2} \times 2}{73000} = \frac{209328}{73000} = \begin{matrix} \text{£} & \text{s.} & \text{d.} \\ 2 & 17 & 4 \end{matrix} \text{ at } 3\frac{1}{2} \text{ per cent.}$$

NOTE.—The fractional difference, at $4\frac{1}{2}$ per cent. is occasioned by neglecting the remainder at 5 per cent.

EXAMPLES FOR PRACTICE.

1. What is the interest of £325 6s. 8d. for 73 days, at 5 per cent. per annum? *Ans* £3 5s. 0 $\frac{3}{4}$ d.
2. What is the interest of £465, for 40 days, at 5 per cent per annum? *Ans*. £2 10s. 11 $\frac{1}{2}$ d.
3. What is the interest of £255 for 80 days at $4\frac{1}{2}$ per cent. per annum? *Ans*. £2 10s. 3 $\frac{1}{2}$ d.
4. What is the interest of £312 for 64 days at $4\frac{1}{2}$ per cent. per annum? *Ans*. £2 11s. 11 $\frac{1}{2}$ d.
5. What is the interest of £127 12s. 6d. for 321 days at 5 per cent. per annum? *Ans*. £5 12s. 2 $\frac{3}{4}$ d.
6. What is the amount of £74 15s. 6d. for 123 days, at $3\frac{1}{2}$ per cent. per annum? *Ans*. £75 13s 1 $\frac{1}{2}$ d.
7. Lent my friend £125 15s. 5d. on the first of January, 1840; how much was due for principal and interest, on the 29th of July following? *Ans*. £129 7s. 9 $\frac{1}{4}$ d.
8. Bought 30 yards of cloth on the 15th of January, at 14s. per yard, and sold it on the 15th of July, at 16s. 6d. per yard, How much do I gain by the whole? *Ans*. £3 4s. 6d.

NOTE.—The answer to this question is obtained by adding 6 month's interest to the first or prime cost.

9. Find the expense of discounting the following Bills of Exchange, viz: one of £250 for 40 days; and one of £260 for 50 days, at 5 per cent. per annum, and $\frac{1}{4}$ per cent. commission. *Ans*. £4 8s. 6d.

10. The following Bills were discounted on the 5th of January;—

A's Bill for £320 due the 10th of February.

B's — for £730 ——— 10th of March.

C's — for £365 ——— 5th of April.

Required the proceeds and discount of each Bill, at 5 per cent. per annum, and $\frac{1}{4}$ per cent. commission.

Ans. { The proceeds of A's £317 12s. 5 $\frac{1}{2}$ d. discount £2 7s. 6 $\frac{1}{2}$ d.
 _____ B's £721 15s. 6d. _____ £8 4s. 6d.
 _____ C's £359 11s. 9d. _____ £5 8s. 3d.

Note.—Bills of Exchange are written orders for the payment of money, at an appointed time. Their forms are various, as are the times of payment. Some are made payable at a certain number of days after date, others at 1, 2, 3, 4, or more months after date, as per agreement between the parties.

In discounting Bills the interest is taken, and not the discount.

RULE. Add the interest of the amount of the Bill for the time it has to run, that is, the time before it becomes due, and the commission into one sum, which subtract from the amount of the bill, and the remainder will be the proceeds, or its present worth.

A Bill of £50 drawn on the 26th of May, payable 3 months after date ; what will be its real value on the 29th of June following ; at 5 per cent. per annum, and $\frac{1}{4}$ per cent. commission being charged for discounting it ?

Here this Bill, including the 3 days grace, will be due on the 29th of August, consequently it will have 2 months to run on the 29th of June.

Bill	...	...	...	...	...	£.	s.	d.
						50	0	0
				s.	d.			
Two months interest	...			8	4	0	10	10
Commission, at $\frac{1}{4}$ per cent	...			2	6			

Value of the Bill on the 29th of June 49 9 2

11. Lent my friend £360, on the 10th of April, which he returned to me as follows :—On the 10th of May, £180 ; on the 4th of July, £80 ; and the balance on the 13th of August. How much is due for interest, at 5 per cent. per annum ?

April 10th, sum due 360 $\times 30 = 10800$
 May 10th, received 180

Sum remaining due, May 10th 180 $\times 55 = 9900$
 July 4th, received 80

Sum remaining due July 4th..... } 100 $\times 40 = 4000$
 which was paid August 13th..... }

7300)24700

Interest required..... £3 7 8

NOTE.—Here the lender wants interest on the sum of £360 from the 10th of April to the 10th of May, viz. 30 days ; then £180 are paid, and £180 remain due on the 10th of May ; on this, interest must be calculated to the 4th of July, being 55 days, when £80 are paid ; on the £100 now remaining due, July 4th ; interest must be calculated to the 13th of August, a period of 40 days.

12. A Bill of £350 was due on the 3rd of March, towards which the following payments were made, viz: on the same day £130, on the 20th of April £120, on the 5th of July £70, and the balance on the 12th of August. How much was due for interest?

Ans. £2 12s. 10½d.

13. A Bill of £350 was due on the 3rd of January; £150 paid the same day, £120 on the 1st of March, and the balance on the 5th of May. Required the interest due to the holder, at 5 per cent. per annum.

Ans. £2 5s. 5½d.

14. If a Bill of £150 be due on the 20th of February, in a leap year, and the following payments be made, viz. on the 5th of March £50, on the 10th of April £60, and the balance on the 4th of June. How much will be due for interest, at 5 per cent. per annum?

Ans. £1 1s. 7½d.

CASE IV.—*To find the interest on account current.*

RULE.—Find the interest on the Dr. and Cr. sides of the account by the 3rd Case, and the difference will be the interest required.

1. What interest is due on the following account on the 31st of December?

DR.		TAYLOR, DAY, AND CO. LONDON.		CR.		
		£.	s. d.		£. s. d.	
May 15.	To Balance.....	185	10 0	July 5.	By Cash.....	200 0 0
June 29	To Goods	140	17 4	Oct. 10.	By Goods.....	150 10 0
Sept. 16.	To Bill due ...	280	6 6	Nov. 18.	By Cash	120 9 6

Calculation of the Interest :

Dr side.

£. s. d.		£. s. d.
185 10 0	× 230 =	42665 0 0
140 17 4	× 185 =	26060 6 8
280 6 6	× 106 =	29714 9 0
		7300)98439 15 8 (13 9 8½

Cr. side.

£. s. d.		£. s. d.
200 0 0	× 179 =	35800 0 0
150 10 0	× 82 =	12341 0 0
120 9 6	× 43 =	5180 8 6
		7300)53321 8 6 (7 6 1

£. s. d.	
13 9 8½	Interest due on the Dr. side.
7 6 1	Interest due on the Cr. side.

6 3 7½ Interest due from Taylor, Day & Co.

NOTE.—In business, when computing interest on accounts or bills of exchange for days, it is customary to omit the odd shillings if under 10; but if 10s. or upwards, to take them as £1. See the following solution.

Or thus:	<i>Dr. side.</i>	<i>Cr. side.</i>
186 × 230 =	42780	200 × 179 = 35800
141 × 185 =	26085	151 × 82 = 12382
280 × 106 =	29680	120 × 43 = 5160
	7300)98545	7300)53342
	13 9 11½	7 6 1½

£. s. d.
13 9 11½
7 6 1½

6 3 10¼ Interest due from Taylor, Day and Co.

2. What interest is due to Smith and Co. on the following account, on the 31st of October?

DR.	SMITH AND CO.	CR.	£. s. d.
	£. s. d.		£. s. d.
Jan. 5. To Cash.....	80 0 0	Jan. 21. By Cash	100 0 0
Mar. 21. Do	59 15 0	May 20. Do.	120 10 0
April 28. Do	70 8 0	June 25. Do	140 8 0
	<i>Ans. £2 2s. 10d. due to Smith and Co.</i>		

CASE V.—When the amount, rate per cent., and time, are given, to find the principal.

RULE.—As the amount of £100 for the given rate and time is to £100, so is the given amount to the principal required.

EXAMPLES.

1. What principal will amount to £384 in 5 years, at 4 per cent. per annum?

Here $4 \times 5 = 20$. 20 is $\frac{1}{5}$ 100

20 Int. of £100 for 5 years.
100

120 Amount of ditto.

As £120 : £100 :: £384 : £320 *Ans.*

2. What sum will amount to £862 10s. in 5 years, at 5 per cent. per annum ? *Ans.* £690.

CASE VI.—*When the amount, principal, and rate per cent. are given, to find the time.*

RULE.—As the interest of the principal for 1 year is to 1 year, so is the whole interest to the required time.

EXAMPLES.

1. In what time will £320 amount to £384, at 5 per cent. per annum ?

$$5 \text{ is } \frac{1}{20}) 320$$

$$\begin{array}{r} \text{£16 Interest of the principal for 1 year.} \\ \text{£384} \\ \text{320} \\ \hline \end{array}$$

£64 whole interest.

As £16 : 1 year :: £64 : 4 years. *Ans.*

2. In what time will £425 amount to £484 10s. at 4 per cent. per annum ? *Ans.* $3\frac{1}{2}$ years.

CASE VII.—*When the amount, principal, and time are given, to find the rate per cent.*

RULE.—As the principal is to the interest, so are £100 to the interest of £100 for the given rate and time; which divide by the given time, and the quotient will be the rate per cent.

EXAMPLES.

1. At what rate per cent. will £320 amount to £384 in 4 years ?

$$\text{£384} - \text{£320} = \text{£64 Interest.}$$

As £320 . £64 :: £100 : £20, Int. of £100 for 4 years.

Then $20 \div 4 = 5$ per cent. *Ans.*

2. A person put out £480 at a certain rate per cent. and received, at the end of 6 years, for principal and interest, £613 4s. Required the rate per cent. *Ans.* £4 12s. 6d.

COMMISSION AND BROKERAGE.

Commission, or Brokerage, is an allowance made to bankers, agents, or brokers, for exchanging money, transacting business, &c., and is generally at so much per cent., without reference to time.

The allowance made to brokers and commission agents varies according to the nature of the business they have to transact. In some cases, 2s. 6d. per cent. is charged; in others, more. Bankers charge 5s. or $\frac{1}{4}$ per cent.

Commission agents usually charge $2\frac{1}{2}$ per cent. when they do not guarantee the debts; but 5 per cent. is their usual charge, when they guarantee, or become responsible for the payment of the debts they contract.

CASE I.—*When the commission or brokerage is at 1 or more than 1 per cent.*

RULE.—This is the same as finding the interest for 1 year. (See Case 1st. in interest.)

EXAMPLE.

Sold goods for James Jones and Son, to the amount of £467 15s.; what do I want for my commission at 5 per cent.

$ \begin{array}{r} \text{£. } s. \\ 5 \text{ is } \frac{1}{20}) 467 \ 15 \\ \hline \text{Ans. } \quad 23 \ 7 \ 9 \end{array} $	$ \begin{array}{r} \text{£. } s. \\ \text{Or, } \quad 467 \ 15 \\ \quad \quad 5 \\ \hline 23,38 \ 15 \\ \quad 20 \\ \hline 7,75 \\ \quad 12 \\ \hline \text{Ans. } \quad \text{£}23 \ 7 \ 9. \end{array} $
--	--

CASE II.—*When the rate per cent. is less than £1.*

RULE.—Divide the given sum by the aliquot parts of a £. contained in the rate per cent. This result divided by 100, will give the commission or brokerage required.

Or, divide the given sum by 100, and the quotient will be the commission at £1 per cent. of which, take aliquot parts for the given rate per cent.

Or, as £100 are to the rate per cent, so is the given sum to the commission required.

NOTE—In some cases, Agents have casual expenses to pay, such as cartage, wharfage, &c. which are chargeable to the party consigning the goods, and must be added to the commission.

EXAMPLES.

1. What is the commission on £1356 at 7s. 6d. per cent.?

s.	£.		£.
5 is $\frac{1}{4}$)	1356	or	13,56
s. d.	—		20
2 6 is $\frac{1}{2}$)	339		—
	169 10		11.20
	—		12
	5,08 10		—
	20		2,40
	—		4
	1.70		—
	12		1.60
	—		—
	8,40		
	4		
	—		
	1.60		
	—		

Ans. £5. 1s. 8½d.

	£.	s.	d.
5s. is $\frac{1}{4}$ d.	13	11	2½
	—	—	—
2s. 6½d. is $\frac{1}{2}$	3	7	9½
	1	13	10¾

5 1 8½ Ans.

Or, as £100 : 7s. 6d. :: £1356 : £5 1s. 8½d. Ans.

2. Sold a quantity of goods for £567, paid 12s. 6d. for wharfage, and £1. 7s. 6d. for cartage. What sum shall I have to remit to my employer, if they allow me 5 per cent. for commission?

s.	£.	
5 is $\frac{1}{20}$)	567	
	—	
	28 7	Commission
	12 6	Wharfage
	1 7 6	Cartage
	—	
	30 7 0	Amount to deduct
	—	
	536 13	Sum to remit

3. What is the commission on £75 12s. 6d. at 7s. 6d. per cent? *Ans.* 5s. 8d.

4. What is the commission on £156 18s. 4d. at 7s. 9½d. per cent? *Ans.* 12s. 3d.

5. What is the Broker's commission on £742 14s. 9d. at 13s. 4d. per cent? *Ans.* £4 19s. 0½d.

6. Required the commission on £568 18s. 4d. at 17s. 6d. per cent. *Ans.* £4 19s. 6½d.

7. What is the Banker's commission on £3560 at 5s. or ½d. per cent? *Ans.* £8. 18s.

8. Consigned goods to the amount of £755 12s. 6d. How much is due for commission at 2½ per cent? *Ans.* £18 17s. 9½d.

9. Rendered an amount of goods consigned to me, which I sold for £756. 17s. 6d., paid for freight, cartage, portorage, &c. £25. 12s. 6d. What is the neat produce of the sale, if I be allowed ¼. 12s. 6d. per cent. for my commission? *Ans.* £696 4s. 10½d.

10. Shipped goods for Thos. Young and Co., to the amount of £365 6s. 8d.; charges of packing, cartage, &c. £2 12s. 6d.; wharfage, entry bond, and shipping, £7 3s. 4d.; commission, 3 per cent. What is the amount of the invoice? *Ans.* £386 1s. 8½d.

11. I am allowed 5 per cent. for commission, and guarantee the debts to my employers. Now, if I do business to the amount of £21656, what will be my neat profit, if I lose by failures £521 12s. 8d., and the necessary expenses for conducting the same be £179 15s. 6d.? *Ans.* £381 7s. 10d.

12. A commission agent, in one year, does business to the amount of £24674 10s., at 5 per cent. Now, suppose him to make 3 bad debts, one of £156 12s. 6d., for which he recovers 7s. 6d. in the £; one of £576. 10s. 6d., for which he recovers 6s. 8d. in the £.; and the other of £184 4s. 6d., for which he only recovers 3s. 4d. in the £.; clerk's salary, £100.; two warehousemen, each 20s. per week; rent and taxes £52 12s. 6d., and other incidental expenses, to the amount of £35. What will be his year's income? *Ans.* £306 6s. 9½d.

INSURANCE.

INSURANCE is a contract of indemnity, by which one party agrees, for a stipulated sum, to insure another party against risk or loss by fire, water, &c.

The party who takes the risk, is the insurer or underwriter ; and the party thus protected, the insured.

The premium or rate per cent. of insurance varies according to the risk, from 1s. 6d. per cent. upwards.

The duty paid to government on all sums insured, is 3s. per cent. The stamp on the paper, called the policy, is 1s. for each sum insured, on inland insurance.

Farming stock is exempt from these duties.

The stamp duty on policies of insurance from one part of the United Kingdom to another is 2s. 6d. per cent.

Covering insurance is when the value of the property, with the premium and other expenses, are insured.

CASE I.—*To calculate inland insurance.*

RULE.—Add the premium and duty into one sum, and proceed as in the rule for Commission. Add one shilling, the cost of the stamp on the policy, to this result, and the sum will be the insurance required.

EXAMPLES.

1. Insured my house for £950, for which I paid 2s. per cent. premium, and 3s. per cent. duty ; stamp on the policy 1s. Required the whole charge.

s.	£.	£.	s.	d.	
2	5 is $\frac{1}{4}$)950	2	7	6	Prem. and duty.
3			1	0	Policy.
	2,37 10				
5	20	2	8	6	Ans.
	7,50				
	12				
	6,00				

2. A farmer insures £950 on his stock, at 2s. per cent.; what does it cost him?

$$\begin{array}{r}
 \text{£.} \\
 2 \text{ is } \frac{1}{100}) 950 \\
 \hline
 .95 \\
 20 \\
 \hline
 19.00
 \end{array}$$

Ans. 19s.

NOTE.—The duty and policy, in the following questions, are the same as in the first Example, if not otherwise expressed.

3. What is the insurance on £456 15s. the premium being 7s. 6d. per cent?

Ans. £2 8s. 11½d.

4. Insured goods to the amount of £360, premium 5s. per cent. Required the expense.

Ans. £1. 9s. 9½d.

5. Insured £750, premium 10s. per cent. Required the expense.

Ans. £4 18s. 6d.

6. Insured £750, premium 5s. per cent, policy 2s. 6d. per cent, and duty 3s. per cent. Required the expense.

s.	d.		s.	£.
5	0	Premium.	10 is ½	(720
2	6	Policy	d.	—
3	0	Duty.	6 ½	375
				18 15
10	6			3.93 15
				20
				18.75
				12
				9.00

7. Insured £1750, premium 2½ per cent., policy 2s. 6d. per cent, duty 3s. per cent., and ½ per cent. commission on the amount of insurance. Required the expense.

Ans. £48 13s. 8d.

8. Insured £6320, premium 2½, duty 3s. per cent., policy 2s. 6d., per cent. with a return of 10 per cent. on the premium; required the expense of the same.

Ans. £152 9s. 4½d.

CASE II.—*To find the amount to be insured, to include given sum and costs.*

RULE.—As the difference between £100 and the rate per cent. of insurance is to £100 so is the value of the property to the sum to be insured.

EXAMPLES.

1. What sum must be insured to cover £750, the premium being £3 14s. 6d. per cent., duty 3s. per cent., and policy 2s. 6d. per cent?

£.	s.	d.	£.						
3	14	6	100						
	3	0	4						
	2	6	—	£.	£.	£.	s.		
				As 96	: 100	:: 750	: 781	5	<i>Ans.</i>
4	0	0							

2. What sum must I insure to cover £2500, the premium being $2\frac{1}{2}$ per cent., duty 3s. per cent., and policy 2s. 6d. per cent?

Ans. £2568 1s. 0 $\frac{3}{4}$ d

3. Insured a ship and cargo for £6750, which will cover expenses; premium £4 4s. 6d. per cent., duty 3s. per cent., and policy 2s. 6d. per cent. Required the value of the ship and cargo.

Ans. £6446 5s. 0d.

COMPOUND INTEREST.

COMPOUND INTEREST is, when the interest due is not paid, but remains in the borrower's hands, and is added to the principal for a new principal, which bears interest at the given rate per cent. Thus a new principal bearing interest, is formed at the end of each year or time when the interest becomes due.

The last new principal will be the amount. If the first principal or sum put out be subtracted from this amount, the remainder will be the interest for the whole time.

N.B. The interest of each succeeding principal is found as in simple interest.

EXAMPLES.

1. What is the compound interest and amount of £325, for $3\frac{1}{2}$ years, at 5 per cent. per annum?

	£.	s.	d.	
5 is $\frac{1}{20}$	325	0	0	sum lent, or 1st year's principal.
	16	5	0	the 1st year's interest.
5 is $\frac{1}{20}$	341	5	0	the 1st year's amt. or 2nd yr's. prin.
	17	1	3	2nd year's interest.
5 is $\frac{1}{20}$	358	6	3	2nd yr's. amount, or 3rd yr's. prin.
	17	18	$3\frac{1}{2}$	3rd year's interest.
	376	4	$6\frac{1}{2}$	3rd year's amt., or 4th year's prin.
	9	8	$1\frac{1}{2}$	half year's interest.
	385	12	8	Amount.
	325	0	0	principal.
	60	12	8	interest required.

2. What is the compound interest of £350 15s. 5d. for 3 years, at 5 per cent? *Ans.* £55 5s. 9½d.

3. Required the amount of £450 for 5 years at 4 per cent. per annum compound interest. *Ans.* £547 9s. 10½d.

4. Lent £245 15s. 5d. 3 years and 9 months ago; how much is due to me at 5 per cent. per annum compound interest? *Ans.* £295 3s. 6½d.

5. What is the difference between the simple and compound interest of £365, for 3 years and 4 months, at 5 per cent. per annum? *Ans.* £3 14s. 9½d.

6. Required the compound interest on £270, for 3 years and 9 months, at 5 per cent. per annum, the interest being payable half yearly. *Ans.* £54 19s. 1d.

7. A and B, each lent £500, for 3 years, at 5 per cent. per annum, compound interest: by agreement, A's interest was to be paid half yearly, and B.'s yearly. Query, which had the advantage, and how much?

Ans. A received £1 0s. 7½d. more than B.

BUYING AND SELLING STOCK.

THE term Stock is applied to those sums of money lent to Government, which constitute the Funds, or National Debt for which the lenders, or their assigns, receive interest out of the public taxes.

It is also applied to the sums which form the capital of the Bank of England, the East India Company, South Sea Company, &c., called Bank Stock, East India Stock, &c.

The National Debt amounted, on the 5th of January, 1841, to £767,946,051 9s. 1d. This sum has been borrowed, by government, of different individuals and companies, in various sums, called per cents. or hundreds; as the 3 per cents., 3½ per cents., 4 per cents., 4½ per cents., and 5 per cents. The terms on which these different Stocks were taken or bought, are as follows:—A specific sum was given for a nominal one; thus the price of the 3 per cents. was £60 for £100 stock, which yields a dividend of £3 a year; for the 3½ per cents., £70 were given for the £100 stock, which yields a dividend of £3 10s. a year; for the 4 per cents., £80 were given for the £100 stock, which yields a dividend of £4 a year, and so on, for the rest; consequently, for all sums originally invested in government securities, 5 per cent. per annum, is paid. The 4, 4½, and 5 per cent Stocks have been reduced to 3½ per cent. Stock, or paid off at par, at the option of the holder.

The Debts of Government and Joint Stock Companies differ from other contracts, inasmuch as the public creditor or stock-holder can only claim his interest; he may, however, sell or transfer his claim to any other person, and thus obtain his capital, more or less as the price of Stock may be; this is commonly called Stock Jobbing, in which a great trade is carried on in London.

NOTE—This debt was commenced in the reign of William the Third, in order to carry on an expensive war with France, and has been increased from time to time, for the same purpose, to the above incredible amount!! Besides the above sum, which is called the Funded Debt, there is the Unfunded Debt, amounting to upwards of 33 millions. The forms of this debt are various; such as Exchequer Bills, Navy Bills, &c., issued under the sanction of an Act of Parliament, to meet contingent expenses, for which no provision has been made. These Bills bear interest, at a certain rate per cent. per day, and are distributed among those who are willing to advance their value. After a certain time they are received in payment by Government for taxes, and the interest due on them at the same time. The interest of the Funded Debt, January 5th, 1841, was £24283940 11s. 5d.

The National Debt, though stated to be £767,946,051 9s. 1d. is not sterling money, but stock ; as the 3 per cents., $3\frac{1}{2}$ per cents. &c. And were any of these stocks paid off, the holder of £100, 3 per cent. stock, would only receive £60 ; the holder of £100, $3\frac{1}{2}$ per cent. stock, £70 ; and so on.

The Bank of England, East India Company, South Sea Company, and all other Companies, where the trading capital is raised in shares, are Joint Stock Companies. The Bank Stock is divided into shares of £200 each. The Shares in other Companies vary, and are as agreed upon by the founders. The shares of many of our Joint Stock Banks, Insurance Companies, &c., are as low as £5 each, and are all transferable.

CASE I.—*To find the cost of any given quantity of stock.*

RULE.—Add the broker's charge to the price per cent when a purchase is made, but subtract it in case of a sale ; then, as £100 stock is to the sum in the first case, or the difference in the second, so is the given quantity of stock to the proceeds required.

Note.—Every purchase or sale of stock is made through the medium of a broker, whose charge is 2s. 6d., or $\frac{1}{4}$ for every £100 of stock. A person purchasing stock through the medium of a country banker, will be subject to an additional charge of 5s. per cent. upon the sum invested, which must be added to the $\frac{1}{4}$ per cent.

EXAMPLES.

1. Bought £500 of $3\frac{1}{2}$ per cent. stock, at 95 per cent. ; what does it cost me ?

£95

0 2 6

			<i>Stock.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	
As £100	:	95 2 6	::	£500	:	475 12 6	the cost required.

2. Sold £800 of 3 per cent. stock, at $74\frac{1}{2}$ per cent. Required the amount of the proceeds.

$74\frac{1}{2}$
 $\frac{1}{8}$

$74\frac{1}{8}$ or $74\frac{1}{4}$.

As £100 : £ $74\frac{1}{4}$:: £800 *Stock* : £594 *Ans.*

3. Bought £625 of $3\frac{1}{2}$ per cent. stock, at $79\frac{1}{4}$ per cent ; to what does it amount ? *Ans.* £496 1s. 10 $\frac{1}{2}$ d.

4. Sold £750 stock, in the $3\frac{1}{2}$ per cents. at $85\frac{1}{2}$ per cent. ; required the proceeds. *Ans.* £641 5s.

5. What is the cost of £1500 stock, at 105 per cent.?

Ans. £1576 17s. 6d.

6. Bought £1200 stock. at $84\frac{5}{8}$ per cent., and sold it for $85\frac{1}{4}$ per cent. How much do I gain by this transaction?

Ans. £4 10s.

7. Bought £5000 stock, at $95\frac{1}{4}$ per cent., and sold it at $93\frac{1}{4}$ per cent. How much do I lose?

Ans. £106 5s.

CASE II.—*To find what quantity of stock can be purchased with a given sum of money.*

RULE.—Add $\frac{1}{8}$ per cent. to, or subtract it from, the given rate, according as the stock is bought or sold; then, as the sum or difference, as the case may be, is to £100 stock, so is the given sum to the amount of stock required.

EXAMPLES.

1. How much 3 per cent. stock, at $85\frac{7}{8}$ per cent., can I purchase for £516?

$$\begin{array}{r} £85\frac{7}{8} \\ \frac{1}{8} \\ \hline \end{array}$$

As 86 : £100 Stock : £516 : £600 Stock.

2. What quantity of $3\frac{1}{8}$ per cent. stock, at $94\frac{1}{8}$ per cent., must be sold to produce £940?

$$\begin{array}{r} £94\frac{1}{8} \\ \frac{1}{8} \\ \hline \end{array}$$

As 94 : £100 Stock :: £940 : £1000 Stock.

3. What quantity of $3\frac{1}{2}$ per cent. stock, at $94\frac{1}{4}$ per cent., can be purchased for £755?

Ans. £800 stock.

4. What quantity of stock, at $85\frac{1}{4}$ per cent., must a person sell to produce £1021 10s.?

Ans. £1200 stock.

5. What amount of $3\frac{1}{2}$ per cent. stock, at $82\frac{1}{8}$ per cent., can I purchase for £1245 sterling; if the broker's commission be 2s. 6d. per cent., and the banker's 5s. per cent.?

Ans. £1500 stock.

CASE III.—*To find what sum must be laid out in a given stock, to produce a given yearly income.*

RULE.—As the rate of interest the stock bears is to £100 stock, so is the yearly income to the amount of stock to be purchased; then find the value of this stock by Case 1st, which will be the answer.

EXAMPLES.

1. What sum of money must be laid out in the $3\frac{1}{2}$ per cents., at $89\frac{7}{8}$ per cent., to produce an income of £350 a year?

As £3½ : £100 Stock :: £350 : £10000 Stock.

$$\begin{array}{r} £89\frac{7}{8} \\ \frac{1}{8} \\ \hline \end{array}$$

As £100 Stock : 90 :: £10000 Stock : £9000 Sterl., Ans.

2. What sum must I invest in the 3 per cents., at $78\frac{1}{4}$ per cent., to produce a yearly income of £366?

Ans. £8033 10s.

CASE IV.—To find the rate per cent. which money produces when invested in the funds.

RULE.—As the sum of the given price and broker's commission is to the interest the stock bears, so is £100 to the rate per cent.

EXAMPLES.

1. If $3\frac{1}{2}$ per cent. stock sell at $97\frac{7}{8}$ per cent; what rate per cent. shall I make of my money, by investing it in this stock?

$$\begin{array}{r} £97\frac{7}{8} \\ \frac{1}{8} \\ \hline \end{array}$$

As 98 : £3 10 :: £100 : £3 11s. $5\frac{1}{2}$ d. per ct. Ans.

2. If I invest money in the $3\frac{1}{2}$ per cent. stocks, at $69\frac{7}{8}$ per cent.; what rate of interest shall I have for my money?

Ans. £3 13s. $10\frac{1}{2}$ d. nearly.

DISCOUNT.

DISCOUNT is an allowance made for the payment of money before it is due.

The present worth of any sum of money due at the expiration of a given period of time, is such a sum, as if put out to interest, at the rate per cent. the discount is to be made, would in this time amount to the given sum or debt.

NOTE.—Where no given rate per cent. is mentioned, 5 per cent. is always understood.

RULE.—As the amount of £100 for the given rate and time is to the interest, so is the given sum or debt to the discount required.

Or, as the amount of £100, for the given rate and time, is to £100, so is the given sum or debt to the present worth required.

NOTE.—The difference between the sum or debt and the discount, is the present worth; and the difference between the sum or debt and the present worth, is the discount.

EXAMPLES.

1. Bought a quantity of goods for £500, with which I am to have 6 months credit. How much ready money will discharge the same, if I be allowed 5 per cent. per annum discount?

Or, what is the present worth of £500, due 6 months hence, at 5 per cent. per annum discount?

m. £. s.
6 is $\frac{1}{4}$) 5 0 Interest of £100 for 1 year.

2 10 Ditto ditto for 6 months.
100

102 10 Amount of £100 for 6 months.

£. s. £. s. £. £. s. d.
As 102 10 : 2 10 :: 500 : 12 3 10 $\frac{1}{4}$, the discount.

£. s. d.
500 0 0 the given sum.
12 3 10 $\frac{1}{4}$ discount.

487 16 1 $\frac{1}{4}$ the present worth.

£. s. £. £. £. s. d.
Or, As 102 10 : 100 :: 500 : 487 16 1 $\frac{1}{4}$ present worth.

NOTE.—If £487 16s. 1 $\frac{1}{4}$ d. were put out to interest at 5 per cent. per annum, for 6 months, it would amount to £500; consequently, the person would receive £500 at the expiration of 6 months, according to contract.

2. What is the difference between the discount and interest of £100, for 12 months, at 5 per cent. per annum?

	£.	£.	£.	£.	s.	d.	
As	105	:	5	::	100	:	4 15 2½ the discount.
					5	0 0	the interest.

4 9½ the difference.

NOTE.—In business, the interest is generally taken for the discount, which is incorrect. For example, if a person sell a quantity of goods for £100, subject to 12 months credit, were to allow £5 for ready money, he would only receive £95. Now place this £95 out on interest, at 5 per cent. per annum, for 12 months, it would amount to £99 15s. ; consequently, he would not receive his £100 as per contract.

3. Bought a quantity of goods for £550, with which I am to have 9 months credit. How much ready money will pay the same, if I be allowed 5 per cent. per annum discount?

Ans. £530 2s. 4½d.

4. Bought goods to the amount of £400, with which I am to have 12 months credit. How much must I receive, at 5 per cent. per annum, for discount, if I pay ready money?

Or, what is the discount of £400, due 12 months hence, at 5 per cent. per annum?

Ans. £19 0s. 11½d.

5. What is the present worth of £420 15s.. due 8 months hence, at 4 per cent. per annum?

Ans. £409 16s. 5d.

6. Bought goods to the amount of £425 16s. 6d., on the 1st of January, to be paid for on the 1st of May following. How much ready money will discharge the same, allowing 4½ per cent. per annum discount?

Ans. £419 7s. 2½d.

7. An account of £360 will be due on the 5th November next. How much ready money will discharge the same. this day, May 5th, at 6 per cent. per annum discount?

Ans. £349 10s. 3½d.

8. An account of £235 18s. 6d. due on the 21st of December next. How much must I allow for present money, this day, May the 9th, at 6½ per cent. per annum discount?

Ans. £9 2s. 6½d.

9. Bought a quantity of goods for £200 ready money, and sold them for £260, to be paid for 8 months hence. How much do I gain, allowing 5 per cent. per annum discount, for present payment?

Ans. £51 12s. 3d.

10. Bought goods for £450, to be paid for as follows:—
 ¼ at 4 months, ¼ at 8 months, and the remainder at 12 months.
 How much present money will discharge the same, at 4½ per cent. per annum, discount?

Ans. £436 12s. 0½d.

11. An account of £250 17s. 6d., due 15 months hence. How much ready money will discharge the same, at £7 12s. 6d. per cent. per annum discount? *Ans.* £229 0s. 10½d.

12. Bought 460 cwt. at 17s. 6d. per cwt., to be paid for as follows; namely, £130 ready money, and the remainder at two 4 months; that is, one-half at 4 months, and one-half at 8 months. What present money will discharge the same, at 7½ per cent. per annum discount? *Ans.* £392 13s. 9d.

BARTER.

BARTER is the exchanging of one commodity for another.

RULE.—Find the value of that commodity, whose price and quantity are given; reduce this value and the price of the required commodity into the same denomination; divide the former by the latter, and the quotient will be the quantity of the required commodity.

When part cash is paid, or a certain quantity of goods at a given price is taken, subtract the cash, or the value of the goods, as the case may be, from the value of the given commodity, and proceed as above.

EXAMPLES.

1. How many yards of silk, at 3s. 4d. per yard, may I give for 60 yards of cloth, at 12s. 6d. per yard?

10s. is ½	)60 at 12s. 6d.	3s. 4d.
		12
2s. 6d. is ¼	30	
	7 10	40
	37 10	
	20	
	750	
	12	
	4,0)900,0	

225 yards of silk. *Ans.*

Or, by the Rule of Three Inverse.

As 12s. 6d. : 60 yds. : : 3s. 4d. : 225 yards of silk.

2. How many yards of cotton, at 10d. per yard, besides £12 6s. 4d, in cash, must I receive for 50 yards of cloth, at 12s. 6d. per yard?

10s. is $\frac{1}{2}$)50 at 12s. 6d.

2s. 6d. is $\frac{1}{4}$ 25
6 5

31 5 value of the cloth.
12 6 4 received in cash.

18 18 8 sum to be received in cotton.
20

378
12

10)4544

454 yds. 1 qr. $2\frac{3}{4}$ nails of cotton.

3. A and B barter; A has 72 yards of cloth at 15s. per yard, for which B gives him 6 cwt. 2 qrs. of sugar, at £3 12s. per cwt., and the remainder in tea, at 7s. 6d. per lb. How many lbs. of tea must A receive?

10s. is $\frac{1}{2}$)72 at 15s. 2qr. is $\frac{1}{2}$ £3 12s.

5s. is $\frac{1}{4}$ 36

18

21 12

1 16

54 value of the cloth

23 8

23 8 value of the sugar.

s. d. — lb. lb. oz.

As 7 6 : 30 12 :: 1 : 81 9 $\frac{3}{4}$ Ans.

4. A has cloth at 7s. 6d. per yard, ready money, but in Barter he advances it to 8s. B has stuff pieces worth 48s. ready money. How must B charge his goods when he barter A's cloth?

As 7s. 6d. : 8s. :: 48s. : £2 11s. 2 $\frac{1}{2}$ d. Ans.

5. Bartered 200 yards of cloth, worth 15s. per yard, for cotton print, at 9d. per yard. How many yards of cotton had I to receive?

Ans. 4000 yards.

6. Exchanged 598 yards of silk, worth 2s. 6d. per yard, for cotton pieces, at 11s. 6d. per piece. How many pieces did I receive?
Ans. 130 pieces.

7. Sold goods for £125 12s. 6d. and received on account £100. How many yards of silk, at 2s. 6d. per yard, may I receive for the balance?
Ans. 205 yards.

8. Sold goods for £225 16s. and received £150 6s. ready money, and the balance in cloth, at 3s. 4d. per yard. How many yards of cloth had I?
Ans. 453 yards.

9. Bartered cloth at 8s. 6d. per yard, which I sold at 7s. 4d. per yard, ready money; for pilot cloths, worth 4s. 6d. per yard, ready money. Required the barter price, per yard, of the latter.
Ans. 5s. 2½d.

10. Bartered cloth at 12s. 6d. per yard, which I sold at 10s. 6d. ready money; for silk, at 4s. per yard. Required the ready money price per yard of the silk.
Ans. 3s. 4½d.

11. Bought 6 cwt. 3 qrs. 21 lb. of sugar, at £2 16s. per cwt. for which I am to pay $\frac{2}{3}$ in cash, and the remainder in soap, at 4½d. per lb. Required the particulars of my payment.
Ans. £12 19s., and 345½ lb.

12. Bartered 120 yards of cloth, 2½ yards of which were worth £1. for two sorts of coffee, and to have an equal number of lbs. of each, the one worth 2s. per lb. and the other 2s. 4d. Required the number of lbs. of each sort.
Ans. 221 $\frac{7}{11}$ lb. of each.

EQUATION OF PAYMENTS:

EQUATION OF PAYMENTS is to find the mean time when two or more sums of money, due at different dates, may be paid at one payment.

RULE.—Multiply each sum by the time it has to run before it becomes due; divide the sum of these products by the sum of the payments, and the quotient will be the time required, in the same denomination the multiples are taken.

NOTE.—If the times of payments be of different denominations, they must be reduced to the same denomination.

EXAMPLES.

1. A owes B £150 whereof £50 are to be paid at the end of 3 months, £50 at 4 months, and the remainder at 6 months. What time may the whole be paid at once without prejudice to either?

$$\begin{array}{r}
 50 \times 3 = 150 \\
 50 \times 4 = 200 \\
 50 \times 6 = 300 \\
 \hline
 150 \qquad 15,0)65,0(4 \text{ months, } 10 \text{ days, the time required.} \\
 \qquad \qquad 60 \\
 \qquad \qquad \hline
 \qquad \qquad 5 \\
 \qquad \qquad 30 \\
 \qquad \qquad \hline
 \end{array}$$

15)150(10 days.

2. A owes B £300 to be paid as follows; £120 at 3 months, £120 at 6 months, and the remainder at 8 months. Required the equated time to pay the whole. *Ans. 5 m. 6 days.*

3. I hold 3 Bills as follows; one for £50 due 2 months hence, one for £70 due 4 months hence, and the 3rd. for £100 due 5 months hence. At what date must one bill be drawn for the amount? *Ans. 4 months.*

4. Sold goods for £360 to be paid for as follows; £100 at 25 days, £200 at 36 days, and the balance at 95 days. At what time may the whole be paid without loss to either party? *Ans. 42 $\frac{7}{8}$ days.*

5. Bought goods on the 1st of January to the amount of £320 to be paid for as follows; on the 4th of March £100., on the 5th of April £120. and the balance on the 8th of June. At what date may one bill be given for the amount?

Ans. 104 days.

PROFIT AND LOSS.

THIS division of arithmetic enables the tradesman to find his gain or loss per cent. by any sale; what price to fix upon his goods, so as to gain or lose any given rate per cent.; how to find the prime cost, when the selling price and rate per

cent. are given ; how to regulate the selling price, in order to gain or lose a certain rate per cent., and allow a given discount for cash, &c.

NOTE.—Where no rate per cent. for discount, &c. is given, 5 is always understood.

CASE I.—*When the prime cost and selling price are given, to find the gain per cent.*

RULE.—As the prime cost is to the difference between it and the selling price, so are £100 to the gain per cent.

N.B. If the selling price be less than the prime cost, the result or fourth number will be the loss per cent.

EXAMPLES.

1. Bought cloth at 12s. 6d. per yard, and sold it for 15s. per yard. How much do I gain per cent. ?

s.	d.
15	0
12	6

As 12s. 6d. : 2 6 :: £100 : £20 per cent.

2. Bought cloth at 15s. per yard, and sold it for 12s. 6d. per yard ; how much do I lose per cent. ?

s.	d.
15	0
12	6

As 15s. : 2 6 :: £100 : £16 13 4d. loss per cent.

CASE II.—*When the prime cost and gain per cent. are given, to find the selling price.*

RULE.—As £100 is to the sum of £100 and the gain per cent., so is the prime cost to the selling price.

N.B. If it be required to find the selling price, so as to lose a given rate per cent., the rate per cent must be subtracted from £100, instead of being added, for the second term.

Or, take aliquot parts with the rate per cent. of the prime cost, which add to, or subtract from the same, as the case may require. in order to find the selling price.

EXAMPLES.

1. Bought cloth at 16s. 6d. per yard; how must I sell it to gain $12\frac{1}{2}$ per cent.?

$$\begin{array}{rcl} \text{£}100 & \text{Or thus,} & 12\frac{1}{2} \text{ is } \frac{1}{8})16\text{s. 6d.} \\ 12 \ 10 & & \underline{2 \ 0\frac{3}{4}} \end{array}$$

As $\text{£}100 : 112 \ 10 :: 16\text{s. 6d.} : 18\text{s. 6}\frac{3}{4}\text{d. Ans. } 18 \ 6\frac{3}{4} \text{ Ans.}$

2. Bought cloth at 16s. 6d. per yard; how must I sell the same so as to lose $12\frac{1}{2}$ per cent.

$$\begin{array}{rcl} \text{£}100 & \text{Or thus,} & 12\frac{1}{2} \text{ is } \frac{1}{8})16\text{s. 6d.} \\ 12 \ 10 & & \underline{2 \ 0\frac{3}{4}} \end{array}$$

As $\text{£}100 : 87 \ 10 :: 16\text{s. 6d.} : 14\text{s. 5}\frac{1}{4}\text{d. Ans. } 14 \ 5\frac{1}{4} \text{ Ans.}$

CASE III.—When the selling price and gain per cent. are given, to find the prime cost.

RULE.—As the sum of $\text{£}100$ and the gain per cent. is to $\text{£}100$, so is the selling price to the prime cost.

N.B. If the selling price be to lose a certain rate per cent., the difference between $\text{£}100$ and the loss per cent. must be taken for the first term in the stating, in order to find the prime cost.

Or, as the selling price of $\text{£}100$ is to $\text{£}100$, so is the selling price of the commodity to its prime cost.

EXAMPLES.

1. Sold cloth at 15s. 7d. per yard, and by so doing I gained 10 per cent.; what did it cost me?

$$\begin{array}{r} \text{£}100 \\ 10 \\ \hline \end{array}$$

As $110 : \text{£}100 :: 15\text{s. 7d.} : 14\text{s. 2d. prime cost.}$

2. Sold cloth at 18s. 9d. per yard, and by so doing I lost 10 per cent.; what did it cost me?

$$\begin{array}{r} \text{£}100 \\ 10 \\ \hline \end{array}$$

As $90 : \text{£}100 :: 18\text{s. 9d.} : 20\text{s. 10d. prime cost.}$

CASE IV.—*When the prime cost, gain per cent., and the discount allowed to the customer for cash, are given, to find the selling price.*

RULE—As the difference between £100 and the discount allowed to the purchaser, is to the sum of £100 and the gain per cent., so is the prime cost to the selling price.

N.B. If it be required to find the selling price, so as to lose a given rate per cent., the difference between £100 and the loss per cent. must be taken, for the second term in the stating.

EXAMPLE.

How must I sell cloth, which cost me 12s. 6d. per yard, in order to gain 5 per cent. and allow to the purchaser a discount of 10 per cent.?

$$\begin{array}{r} \text{£}100 \\ 10 \\ \hline \end{array} \quad \begin{array}{r} \text{£}100 \\ 5 \\ \hline \end{array}$$

As 90 : 105 :: 12s. 6d. : 14s. 7d. the selling price.

Or, 5 is $\frac{1}{20}$ 12s. 6d.
 $\begin{array}{r} 0 \quad 7\frac{1}{2} \\ \hline \end{array}$

13 1 $\frac{1}{2}$ the sum I must receive, in order to gain 5 per cent.

$$\begin{array}{r} \text{£}100 \\ 10 \\ \hline \end{array}$$

As 90 : £100 :: 13s. 1 $\frac{1}{2}$ d.
 $\begin{array}{r} 10 \\ \hline \end{array}$

9)131 3

14 7 selling price.

Proof. 10 is $\frac{1}{10}$ 14s. 7d.
 $\begin{array}{r} 1 \quad 5\frac{1}{2} \\ \hline \end{array}$

13 1 $\frac{1}{2}$ sum I must receive, in order to gain 5 per cent.

The application of this Case being of much importance to every one in trade, the following Table and Examples are given, to facilitate and enable any person, though unacquainted with the rules of Profit and Loss, to fix the selling price of his goods, so as to gain or lose a certain rate per cent.

TABLE.*			
To gain	5 per cent. add $\frac{1}{20}$ of the prime cost to it for the selling price.		
7 $\frac{1}{2}$	add $\frac{1}{20}$ and $\frac{1}{2}$ of this	for	ditto
10	add $\frac{1}{10}$	for	ditto
12 $\frac{1}{2}$	add $\frac{1}{8}$	for	ditto
15	add $\frac{1}{10}$ and $\frac{1}{2}$ of this	for	ditto
17 $\frac{1}{2}$	add $\frac{1}{8}$ and $\frac{1}{20}$ of prime cost	for	ditto
20	add $\frac{1}{5}$	for	ditto
22 $\frac{1}{2}$	add $\frac{1}{5}$ and $\frac{1}{8}$ of this	for	ditto
25	add $\frac{1}{4}$	for	ditto
33 $\frac{1}{3}$	add $\frac{1}{3}$	for	ditto
50	add $\frac{1}{2}$	for	ditto

EXAMPLES.

How must I sell goods which cost me 12s. 6d. per yard ; to gain 5 per cent., 7 $\frac{1}{2}$ per cent., 10 per cent., 12 $\frac{1}{2}$ per cent., 15 per cent., 17 $\frac{1}{2}$ per cent., 20 per cent., 22 $\frac{1}{2}$ per cent., 25 per cent., 33 $\frac{1}{3}$ per cent., and 50 per cent?

5 is $\frac{1}{20}$	s. d. 12 6 0 7 $\frac{1}{2}$ 13 1 $\frac{1}{2}$ to gain 5.	5 is $\frac{1}{20}$	s. d. 12 6 0 7 $\frac{1}{2}$ 0 3 $\frac{1}{4}$ 13 5 $\frac{1}{4}$ to gain 7 $\frac{1}{2}$.	10 is $\frac{1}{10}$	s. d. 12 6 1 3 13 9 to gain 10
12 $\frac{1}{2}$ is $\frac{1}{8}$	s. d. 12 6 1 6 $\frac{3}{4}$ 14 0 $\frac{1}{4}$ to gain 12 $\frac{1}{2}$.	10 is $\frac{1}{10}$	s. d. 12 6 5 is $\frac{1}{2}$ 1 3 0 7 $\frac{1}{2}$ 14 4 $\frac{1}{2}$ to gain 15.	12 $\frac{1}{2}$ is $\frac{1}{8}$	s. d. 12 6 5 is $\frac{1}{20}$ 1 6 $\frac{3}{4}$ 7 $\frac{1}{2}$ 14 8 $\frac{1}{2}$ to gain 17 $\frac{1}{2}$
20 is $\frac{1}{5}$	s. d. 12 6 2 6 15 0 to gn.	20 is $\frac{1}{5}$	s. d. 12 6 2 $\frac{1}{2}$ is $\frac{1}{2}$ 2 6 0 3 $\frac{3}{4}$ 15 3 $\frac{3}{4}$ to gain 22 $\frac{1}{2}$.	25 is $\frac{1}{4}$	s. d. 12 6 3 1 $\frac{1}{2}$ 15 7 $\frac{1}{2}$ to gain 25
33 $\frac{1}{3}$ is $\frac{1}{3}$	s. d. 12 6 4 2 16 8 to gn.	33 $\frac{1}{3}$ is $\frac{1}{3}$	s. d. 12 6 6 3 18 9 to gain 50 per cent.		

* NOTE.—Here 5 per cent is $\frac{1}{20}$ of £100, 10 is $\frac{1}{10}$ of £100, 12 $\frac{1}{2}$ is $\frac{1}{8}$ of £100, &c. &c.

NOTE.—The following remarks will assist the student in solving as well as comprehending the different questions in Profit and Loss:—

CASE I.

- Question 5. Deduct 5 per cent. from 13s. 9d. to find the neat selling price.
 6. The present worth of 13s. 9d. for 6 months, will be the neat selling price.
 7. Find the neat prime cost, by deducting the given discount for cash.
 8. Find the neat cost and selling price, by deducting $7\frac{1}{2}$ per cent., and also 2d. in the £. from the selling price, for cash.
 9. The present worth, in both cases, will be the prime cost and selling price.
 10. Deduct 5 per cent. for cash, from the nominal price, to find neat prime cost.

CASE II.

2. Find the neat prime cost, as in Question 10, Case 1.
 3. Add the cost of finishing to 12s. 6d., for the neat prime cost.
 4. The present worth for 6 months will be the neat prime cost.
 8. In finding the neat prime cost, 10 per cent. must be deducted from the making, dressing, and dying.
 9. Here 5 per cent. must be taken from the nominal selling price, in order to find the neat selling price.

CASE III.

3. Here 5 per cent. must be taken from the 40s., in order to find the real selling price.
 4. Here the present worth of the 50s. will be the real selling price.

CASE IV.

6. Deduct $2\frac{1}{2}$ per cent. from 12s. 6d., to find the neat or true prime cost.

MISCELLANEOUS EXAMPLES.

1. Find the neat prime cost and selling price, and proceed by Rule 1st.
 2. Here the 5 per cent. is deducted from the 10 per cent on 9s. 6d., that is, from the cost of dressing.
 3. The dressing is computed on the whole length.
 4. The dressing is computed on the length returned. In this question, on $25\frac{1}{4}$ yards for each piece.
 6. The present worth of A's debt for 12 months must be taken from the neat selling price to him, and 5 per cent. from B's, for the neat selling price.
 7. Here the selling price is to gain $12\frac{1}{4}$ per cent. Find by Case 3rd the prime cost; from this deduct the known expenses, and the remainder will be the value of the wool.
 10. The present worth of 17s. 6d. for 9 months is taken for the selling price; then proceed as in Question 7th.
 11. Find the prime cost so as to gain $12\frac{1}{4}$ per cent. From this deduct the known charges. The duties, insurance, commission, &c. are taken on the selling price, viz. 25s.
 13. In this question, 5 per cent. per annum interest is laid on the prime cost (16s.), from the 1st of Jan. till it be sold, which gives the prime cost.

14. Here a bill at 3 months, 3 months after delivery, is giving 6 months credit ; consequently the present worth of 30s. for 6 months will be the neat selling price. The duties, freightage, insurance, &c. is taken on the 30s., then proceed as in question 11th.

15 and 16. From the prime cost deduct the amount of duties, freightage, insurance, &c. Find the prime cost undressed, exclusive of duties, &c., and the difference between these will be the price of dressing ; then proceed as in Question 14.

PRACTICAL EXAMPLES UNDER CASE 1st.

1. Bought cloth at 12s. 6d. per yard, and sold it for 13s. 9d. How much did I gain per cent. ? *Ans.* £10 per cent.

2. Bought goods at 12s. 6d. per yard, and sold them at 14s. How much do I gain by laying out £56 ?

Ans. £6 14s 4½d.

3. A merchant sold 239 yards of cloth at 13s. per yard, which cost him 12s. 6d. per yard. Required his gain per cent. and whole profit.

Ans. £4 per cent. ; £5 19s. 6d. whole gain.

4. If I buy goods at 35s. per cwt., and sell them for 32s. 6d., how much do I lose per cent., and what by 37 cwt. ?

Ans. £7 2s. 10½d. per cent. ; £4 12s. 6d. loss.

5. Bought goods at 12s. 6d. per yard, and sold them for 13s. 9d., subject to a discount of 5 per cent. for cash. Required my gain per cent.

Ans. £4 10s.

6. Bought goods at 12s. 6d. per yard, and sold them for 13s. 9d. per yard, to be paid for at the end of six months. Required my gain per cent.

Ans. £7 3s. 4d.

7. Bought 96 yards of cloth, at 12s. 6d. per yard, subject to a discount of 5 per cent. for cash ; and sold it for 13s. per yard. Required my whole gain, and how much per cent.

Ans. £5 8s. whole gain ; £9 9s. 5½d. per cent.

8. Bought 96 yards of cloth, at 12s. 6d. per yard, subject to a discount of 7½ per cent. for cash ; and sold it for 12s. 6d. per yard, but had to allow 2d. in the £ for cash. Required my gain per cent., and how much by the whole.

Ans. £7 4s. 1½d. gain per cent. : £4 whole gain.

9. Bought 96 yards of cloth, at 12s. 6d. per yard, with which I am to have 6 months credit ; and sold it at 13s. 9d. per yard, but had to give 9 months credit. Required my whole gain, and gain per cent.

Ans. £5 1s. 6½d. whole gain, £8 13s. 5½d. per cent.

10. Bought sugar at 50s. per cwt., subject to a discount of 5 per cent. for cash; now if I sell this sugar at 50s. per cwt., required my gain per cent. *Ans. £5 5s. 3d.*

CASE 2.

1. Bought 72 yards of cloth, at 12s. 6d. per yard. How must I sell the same per yard, so as to gain $12\frac{1}{2}$ per cent., and what shall I gain by the whole?

Ans. 14s. 0 $\frac{1}{2}$ d. per yard, £5 12s. 6d. whole gain.

2. Bought 72 yards of cloth at 12s. 6d. per yard, subject to a discount of 5 per cent. for cash. How must I sell the same per yard, so as to gain $12\frac{1}{2}$ per cent., and what shall I gain by the whole?

Ans. 13s. 4 $\frac{1}{2}$ d. per yard, £5 6s. 6d. whole gain.

3. Bought cloth at 12s. 6d. per yard, paid $12\frac{1}{2}$ per cent. for finishing. How must I sell the same so as to gain 10 per cent.?

Ans. 15s. 5 $\frac{1}{2}$ d.

4. Bought 72 yards of silk at 4s. 6d. per yard, with which I am to have six months credit. How must I sell the same per yard, so as to gain $7\frac{1}{2}$ per cent., and what shall I gain by the whole? *Ans. 4s. 8 $\frac{1}{2}$ d. per yard, £1 3s. 8 $\frac{1}{2}$ d. whole gain.*

5. Bought sugar at 74s. 8d. per cwt. How must I retail the same per lb., so as to gain $12\frac{1}{2}$ per cent.?

Ans. 9d.

6. A merchant bought 240 yards of cloth at 12s. 6d. per yard. How must he sell the same per yard, so as to lose 5 per cent., and what will he lose by the whole?

Ans. 11s. 10 $\frac{1}{2}$ d. per yard, £7 10s. whole loss.

7. A merchant paid 4s. 3d. per yard for his cloth making, 2s. 3d. per lb. for the wool, and it takes 2 lbs. of wool to make a yard, dying the same 1s. per lb., dressing and other charges 2s. 4d. per yard. How must he sell this cloth, so as to gain $12\frac{1}{2}$ per cent.?

Ans. 14s. 8 $\frac{1}{2}$ d.

8. A merchant paid 4s. 3d. per yard for his cloth making, 2s. 3d. per lb. for the wool, and it takes 2 lb. of wool to make a yard, dying the same 1s. per lb., dressing and other charges 2s. 4d. per yard. How must he sell this cloth per yard, so as to gain $12\frac{1}{2}$ per cent., if the dyer, dresser and maker allow him 10 per cent. on their account?

Ans. 13s. 9d.

9. Bought goods at 50s. per cwt., on which I laid 10 per cent., but allowed 5 per cent. for cash. Required my gain per cent. *Ans. 4 $\frac{1}{2}$ per cent.*

10. How must I sell the following goods, so as to gain $7\frac{1}{2}$ per cent., and what shall I gain by the whole?

73 yards of silk, which cost 3s. 4d. per yard.

36 yards of Irish cloth 2s. 6d. per yard.

59 yards of cloth 8s. 4d. per yard.

Ans. 3s. 7d. per yard, selling price of the silk.

2 $8\frac{1}{4}$ do do. Irish cloth.

8 $11\frac{1}{4}$ do. do. cloth.

£3 1 $10\frac{1}{2}$ whole gain.

11. A shopkeeper bought 3 ends of cloth, each 24 yards, at 17s. 4d per yard; on which he laid $12\frac{1}{2}$ per cent., but allowed 5 per cent. for cash. How much did he gain per cent., and what by the whole?

Ans. £6 17s. 6d. per cent., £4 5s. $9\frac{1}{2}$ d. whole gain.

CASE 3.

1. If by selling goods at 36s. 6d. per cwt., I gain $6\frac{1}{4}$ per cent.; what do they cost me? *Ans.* £1 14s. 4d.

2. If by selling goods at 12s. 6d. per yard, I lose 5 per cent.; what do they cost me? *Ans.* 13s. $1\frac{1}{2}$ d.

3. If I sell sugar at 40s. per cwt., subject to a discount of 5 per cent., I gain $7\frac{1}{2}$ per cent. What does it cost me?

Ans. £1 15s. $4\frac{2}{3}$ d.

4. If by selling goods at 50s. per cwt., subject to six months credit, I gain 5 per cent.; what do they cost me?

Ans. £2 6s. $5\frac{1}{2}$ d.

5. If by selling goods at 50s. per cwt. subject to six months credit, I lose 5 per cent.; what do they cost me?

Ans. £2 11s. 4d.

6. Sold 120 yards of cloth at 8s. 9d. per yard, subject to a discount of 5 per cent. for cash; by so doing, I lost 10 per cent. What was my whole loss, and what did it cost me per yard? *Ans.* £5 10s. 10d. whole loss. 9s. $2\frac{1}{2}$ d. $\frac{1}{3}$ cost per yd.

7. If by selling goods at 25s. per cwt., I gain $7\frac{1}{2}$ per cent.; what shall I gain per cent., when I sell them for 27s. 6d. per cwt.? *Ans.* $18\frac{1}{2}$ per cent.

8. If by selling goods at 33s. per cwt., I gain 10 per cent.; what must I charge the same, to gain $12\frac{1}{2}$ per cent.?

Ans. £1 13s. 9d.

9. After deducting £2 18s. 4d. per cent. for income tax, and $4\frac{1}{2}$ per cent. from the remainder for local rates, I had £139 1s. $5\frac{1}{2}$ d. left. Required my salary. *Ans.* £150,

CASE 4.

1. How must I sell cloth per yard, which cost me 12s. 6d., so as to gain 10 per cent., and allow the customer 5 per cent. for cash ?

Ans. 14s. 5½d.

2. How must I sell goods which cost me 35s. 6d. per cwt., so as to gain 7½ per cent., and allow the customer 10 per cent. for ready money ?

Ans. £2 2s. 4¾d.

3. Bought cloth at 17s. 6d. per yard. How must I sell the same, in order to gain 8 per cent., and allow the purchaser 4½ per cent. for cash ?

Ans. 19s. 9½d.

4. Sold sugar, which cost me 40s. per cwt., at 6½ per cent. profit. How much must I charge the same, if I allow 5 per cent. for cash, so as to gain my regular profit of 6½ per cent. ?

Ans. £2 4s. 8½d.

5. A merchant bought 360 yards of cloth, at 17s. 6d. per yard. How must he sell the same, per yard, so as to gain 7½ per cent., and allow the customer 7½ per cent. for cash, and what will he gain by the whole ?

Ans. £1 0s. 4d. per yard, £23 12s. 6d. whole profit.

6. Bought cloth at 12s. 6d. per yard, subject to a discount of 2½ per cent. for cash. How must I sell the same, so as to gain 10 per cent., and allow to the customer 5 per cent. for cash ?

Ans. 14s. 1½d.

7. Bought goods at 30s. per cwt. How must I sell the same, so as to lose 10 per cent., and allow 5 per cent. for cash ?

£1 8s. 5⅙d.

MISCELLANEOUS EXAMPLES.

1. Bought sugar at 72s. per cwt., but had 12½ per cent. allowed for cash. Now if I sell this sugar for 72s. per cwt., subject to a discount of 5 per cent. for cash ; how much do I gain per cent. ?

Ans. £8 11s. 5½d. per cent.

2. Bought 16 pieces of cloth, each 24 yards, at 9s. 6d. per yard ; cost of dressing 10 per cent., subject to a discount of 5 per cent. for cash. How must I sell the same per yard, to gain 12½ per cent., and what shall I gain by the whole ?

Ans. 11s. 2d. per yard, £24 6s. 5½d. whole gain.

3. Bought 8 pieces of cloth, each 23¾ yards, at 12s. 6d. per yard. Suppose I have half a yard allowed from each piece for measure ; cost of dressing 10 per cent., subject to 5 per cent. discount for cash. How must I sell the same per yard, to gain 12½ per cent., and what shall I gain by the whole ?

Ans. 15s. 1d. per yard, £15 8s. 9¾d. whole gain.

4. Bought 4 pieces of cloth, each $24\frac{1}{2}$ yards, at 10s. 9d. per yard, subject to an allowance of half a yard from each piece for measure, and $1\frac{1}{4}$ per cent. for cash; cost of dressing $12\frac{1}{4}$ per cent., subject to a discount of 5 per cent. How must I sell the same per yard, so as to gain 10 per cent., and what shall I gain by the whole? Each piece when dressed measured $25\frac{1}{2}$ yards.

Ans. 12s. $5\frac{1}{2}$ d. per yard, £5 15s. $7\frac{1}{2}$ d. whole profit.

5. Sold 120 yards of silk at 3s. 9d. per yard, at a profit of $12\frac{1}{2}$ per cent.; but my customer compounded with his creditors for 6s. 8d. in the £. How much did I lose?

Ans. £12. 10s.

6. Bought 60 yards of cloth at 12s. 6d. per yard, and sold it at a profit of 10 per cent., subject to 12 months credit, or 5 per cent. discount for cash. Now if I sell 30 yards to A, who takes the credit, and 30 yards to B, who takes the discount. How much do I gain per cent. by each, and how much by each?

Ans. 17s. $10\frac{1}{2}$ d. gained by A, and £4 15s. $2\frac{1}{2}$ d. per cent.

16s. $10\frac{1}{2}$ d. gained by B, and £4 10s. 0d. per cent.

7. A merchant received an order for 320 yards of cloth. at 12s. 6d. per yard. Now it will take 2 lb. of wool to make a yard, cost of making 4s. per yard, dressing 2s. per yard, dying the wool 1s. per lb. How much may he give per lb. for his wool, so as to gain $12\frac{1}{2}$ per cent. by this trade, and what sum will he gain by the said order?

Ans. 1s. $6\frac{1}{2}$ d. per lb., £22 4s. $5\frac{1}{2}$ d. whole gain.

8. Bought tea at 4s. 2d. per lb., and sold it at a profit of 5 per cent. How many lb. shall I have to sell, to gain £45?

Ans. 4320 lb.

9. Bought 120 yards of cloth at 12s. 6d. per yard, and sold 40 yards at 14s. 6d. per yard, 50 yards at 13s. 6d. per yard, and 10 yards at 13s. 9d. per yard. How must I sell the remainder per yard, so as to gain $12\frac{1}{2}$ per cent. by the whole?

Ans. 14s. 9d.

10. Received an order for cloth, at 17s. 6d. per yard. Now it will take 2 lb. 4 oz. of wool to make a yard, cost of making 4s. 6d. per yard, dressing, &c 2s. 5d. per yard, dying the wool 1s. per lb. How much may I give per lb. for my wool, so as to gain 10 per cent. if I have to give 9 months credit; supposing the dyer, dresser, and maker to allow me $7\frac{1}{2}$ per cent. on their accounts?

Ans. 3s. 0 $\frac{1}{2}$ d. per lb.

11. Received an order for 10 pieces of cloth, each 24 yards, at 25s. per yard, to be delivered in New York. Now it will take $2\frac{1}{2}$ lb. of wool to make a yard, dying the same 1s. per lb., making 4s. per yard, and dressing 2s. 4d. per yard; duties 25 per cent.; freightage, insurance, commission, &c. 5 per cent. How much may I give per lb. for my wool, so as to gain $12\frac{1}{2}$ per cent., if I allow 2d. in the £. for cash, and the dyer, dresser, and maker allow 5 per cent.: and what shall I gain by the whole?

Ans. 2s. 10d. per lb., £33 1s. $1\frac{1}{4}$ d. whole gain.

12. Bought 1 cwt. of bristles for £12 10s.; when dressed, I had 3 lb. of fine long whites, worth 12s. per lb., and 8 lb. worth 5s. per lb. How must I sell the remainder per lb., to gain 15 per cent. by the whole? Cost of dressing 30s., and waste 3 lb.

Ans. 2s. 6d.

13. Bought 60 cwt., at 16s. per cwt. on the 1st of January, on which I laid $12\frac{1}{2}$ per cent. profit. Now if I sell 20 cwt. on the 1st of March, 20 cwt. on the 1st of June, and the remainder on the 1st of July—required my gain per cent., and how much by the whole?

Ans. £10. 10s. 1d. per cent., £5 2s. 8d. whole gain.

14. Received an order for 480 yards of cloth, at 30s. per yard, to be delivered in the United States, and to be paid for in a bill at three months, 3 months after it was shipped. It will take two lb. of wool to make a yard; dying the same, 9d. per lb., making 4s. per yard, and dressing 2s. 5d. per yard; duties 35 per cent., insurance $3\frac{1}{2}$ per cent., freightage, commission, &c., 27s. 6d. per cent. How much may I give, per lb., for my wool, so as to gain 10 per cent., and what shall I gain by the whole? The dyer, dresser, and maker, allowing 10 per cent. on their accounts.

Ans. 3s. $8\frac{3}{4}$ d. per lb., and £63 17s. 2d. nearly whole.

15. Received an order for cloth at 21s. per yard, to be paid for in a bill at 6 months, 6 months after it is delivered. It will take 2 lb. of wool to make a yard; dying the same, $9\frac{1}{4}$ d. per lb., dressing $12\frac{1}{2}$ per cent., and making 3s. 9d. per yard; duties 20 per cent., insurance $3\frac{1}{2}$ per cent.; freightage, brokerage, and commission, $1\frac{1}{2}$ per cent. How much may I give per lb. for my wool, so as to gain 20 per cent.; if the dyer, dresser, and maker allow 5 per cent on their accounts?

Ans. 2s. $6\frac{1}{4}$ d.

16. Received an order for 480 yards of cloth, at 25s. 2½d. per yard, subject to a discount of 10 per cent., cost of making 4s. 6d. per yard. dressing 12½ per cent., dying the wool 1s. per lb.; and it will take 2 lb. 2 oz. of wool to make a yard: duties 26 per cent., insurance 2 per cent., freightage 1½ per cent., and commission 10s. per cent. How much may I give per lb. for my wool, so as to gain 10 per cent.; the dyer, dresser, and maker, allowing 12½ per cent. on their accounts; and what shall I gain by the whole?

Ans. 2s. 9¼d. per lb., £49 10s. whole gain.

FELLOWSHIP, OR PARTNERSHIP.

FELLOWSHIP OR PARTNERSHIP is, when two or more persons join in company for the purpose of carrying on trade, and agree to share the profits or sustain the losses, as the case may be.

It is divided into simple and compound. Simple fellowship, sometimes called single or fellowship without time, is when each person takes his proportionate share of the gain according to his money or stock in trade, without reference to time.

Compound fellowship, sometimes called double fellowship, or fellowship with time, is when time is taken into account.

CASE 1.—When each partner advances the same sum.

RULE.—Divide the whole gain or loss by the number of partners, and the quotient will be each person's share.

EXAMPLES.

1. A, B, and C, entered into partnership, by which they gained £756 15s. 6d., required each person's share.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 3 \overline{)756 \quad 15 \quad 6} \end{array}$$

252 5 2 each man's share.

2. Four persons entered into partnership, by which they gained £956 16s. 8d., required each person's share.

Ans. £239 4s. 2d.

3. Three persons, each advancing £400, entered into a speculation, by which they gained £360; required each person's share, and gain per cent.

Ans. { £120 each person's share,
30 per cent.

CASE II.—When the business is divided into shares.

RULE.—Divide the whole gain or loss by the whole number of shares, and the quotient will be the value of one share; multiply this share by the number of shares each person holds, and the respective products will be the answer.

EXAMPLES.

1. A mercantile house is divided into thirteen shares, of which A holds 3, B 4, and C the remainder. They gained £2600. Required each person's share.

$$\begin{array}{r} \text{£.} \\ 13 \overline{)2600} \\ \hline 200 \text{ value of one share.} \end{array}$$

$$200 \times 3 = 600 \text{ A's share.}$$

$$200 \times 4 = 800 \text{ B's share.}$$

$$200 \times 6 = 1200 \text{ C's share.}$$

2600 proof

2. A bank consisting of 4 partners, is divided into 25 shares, of which A holds 8, B 6, C 7, and D the remainder. They gained £4000. Required each person's share.

Ans. A's £1280, B's £960, C's £1120, and D's £640.

3. A mercantile house consisting of 3 partners, A, B, and C. A holds $\frac{4}{11}$, B $\frac{6}{11}$, and C the remainder. They gained £750. Required each person's share.

$$\text{Ans. } \left\{ \begin{array}{l} \text{£200 A's share.} \\ \text{£300 B's share.} \\ \text{£250 C's share.} \end{array} \right.$$

CASE III.—When each partner advances a different sum of money, as a stock, to carry on the business.

RULE.—As the whole sum advanced, is to the whole gain or loss, so is each man's sum or stock to his share of the gain or loss.

NOTE.—The gain or loss per cent. may be found as follows: as the whole stock is to the whole gain or loss, so is £100 to the gain or loss per cent.

EXAMPLES.

1. Three persons, A, B, and C, entered into partnership; A advanced £320 6s. 8d., B £500 3s. 4d., and C £725 10s., they gained £756 13s. 4d. Required each person's share.

£.	s.	d.
320	6	8
500	3	4
725	10	0

1546 0 0 the sum or stock.

£.	£.	s.	d.	£.	s.	d.	£.	s.	d.				
As 1546 :	756	13	4	::	320	6	8	:	156	15	7½	³⁰¹⁵⁰ ₃₇₁₀₄₀	A's share.
As 1546 :	756	13	4	::	500	3	4	:	244	15	11½	⁹⁸⁷⁹⁰ ₃₇₁₀₄₀	B's share.
As 1546 :	756	13	4	::	725	10	0	:	355	1	8¼	⁹⁹¹²⁶⁰ ₃₇₁₀₄₀	C's share.
										756	13	4	Proof.

To find the gain per cent.

£.	£.	s.	d.	£.	£.	s.	d.
As 1546 :	756	13	4	::	100	48	18 10½ per cent.

2. Two persons, A and B, entered into partnership ; A advanced £120, and B £360, they gained £260. Required each person's share, and their gain per cent.

Ans. £65 A's share, £195 B's share, £54 3s. 4d. gain per cent.

3. Three persons entered into partnership ; A advanced £325, and B £650, and C £487 10s. ; they gained £1000. Required each person's share.

Ans. £222 4s. 5½d. 975 A's, £444 8s. 10½d. 1950 B's, and £333 6s. 8d. C's.

4. The money and effects of a bankrupt, after paying all expenses, amount to £634 6s. 8d. At this time he stands indebted to A £567 3s. 4d., to B £742 9s. 6d. and to C £593 7s. 2d. What sum will each person receive ?

Ans. £189 1s. 1½d. A's share, £247 9s. 10d. B's share, and £197 15s. 8½d. C's share.

5. Two persons, A and B, entered into partnership ; A advanced £500 17s. 10d., and B £735 ; they gained £300 15s. Required each person's share of the gain.

Ans. £121 17s. 9½d. 195484 A's share, £178 17s. 2½d. 101130 B's share.

6. Three persons, A, B, and C, entered into partnership ; A advanced £750, B £800, and C £1050, their whole gain

in one year was £1650. Now, suppose that A has borrowed £400 of his capital, at 5 per cent., B, £500, and C, £600. Required each man's neat gain by this trade.

Ans. £455 19s. 2½d. 2 rem. A's neat gain, £482 13s. 10d. 16 rem. B's neat gain, £636 6s. 11d. 8 rem. C's neat gain.

CASE IV.—*When one or more of the partners have a certain rate per cent. on the profits, or any given sum allowed for extra time spent in the business.*

RULE.—Subtract the amount of the given per centages, or given sum, for extra services, &c., from the whole gain, and divide the remainder, as in Case 3.

EXAMPLES.

1. Three persons, A, B, and C, entered into partnership; A advanced £500, B £600, and C £400. Now, by agreement, A was to have 5 per cent on the profits, for the extra time he spent in the business. Their whole gain amounted to £1750. Required each person's share.

$$\begin{array}{r} \text{£.} \\ 5 \text{ is } \frac{1}{20} \quad 1750 \quad 0 \\ \hline \quad \quad \quad 87 \quad 10 \end{array}$$

1662 10 the sum to divide.

$$500 + 600 + 400 = \text{£}1500.$$

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{As } 1500 : 1662 \quad 10 :: 500 : 554 \quad 3 \quad 4 \\ \hline \quad \quad \quad \quad \quad \quad \quad 87 \quad 10 \quad 0 \end{array}$$

641 13 4 A's share.

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{£.} \quad \text{£.} \\ \text{As } 1500 : 1662 \quad 10 :: 600 : 665 \end{array}$$

$$\text{As } 1500 : 1662 \quad 10 :: 400 : 443 \quad 6 \quad 8 \text{ C's share.}$$

2. Three persons, A, B, and C, entered into partnership; A advanced £450 6s. 8d., B £325 13s. 4d., and C £800. They gained £860; by agreement, A is to have 4 per cent. on the profits, and B £100, for extra management in the business, and the remainder to be divided according to their respective stocks. Required each person's share.

£.	s.	d.		£.	s.	d.	
207	6	8½		149	18	9½	
34	8	0		100	0	0	
241	14	8½	A's share.	249	18	9½	B's share.
				368	6	5½	C's share.

3. Four persons entered into partnership, A advanced £700, B £800, C 1200, and D £1400, at the end of the year they find their profits to amount to £3000; by agreement, A is to have 2½ per cent. on the profits, and B 3½ per cent. for extra time spent in the business; the remainder to be divided among them in proportion to their respective stocks. Required each person's share.

£.	s.	d.	r.		£.	s.	d.	r.	
481	9	3	36	A's proportion.	550	4	10½	6	B's proportion.
75	0	0		Extra.	105	0	0		Extra.
556	9	3	36	A's share.	655	4	10½	6	B's share.
					825	7	3¾	9	C's share.
					Ans. 962 18 6¼ 31 D's share.				

CASE V.—When one partner has been longer in the trade than another, or when the partners vary their stocks in trade, time is then taken in to consideration, and it is called *Compound or Double Fellowship*.

RULE.—Multiply each person's stock by the time he has it employed in the business; and add these products into one sum; then, as this sum is to the whole gain or loss, so is the product of each man's money and time, to his share of the gain or loss.

EXAMPLES.

Two persons, A and B, entered into partnership, A had £300 in the business 8 months, and B £400 for 9 months; they gained £600. Required each person's share of the gain.

$$\begin{array}{rcl} 300 \times 8 = 2400 & \text{Product of A's money and time.} \\ 400 \times 9 = 3600 & \text{Ditto} \quad \quad \quad \text{B's} \quad \quad \text{ditto.} \end{array}$$

6000 Sum of the products.

£.	£.	
As 6000 : 600 :: 2400	: 240	A's share.
As 6000 : 600 :: 3600	: 360	B's share.

600 Proof.

2. Two merchants, A and B, entered into partnership for 12 months; the first four months A had £500 employed in the business, and the remainder of the time £800, B had £400 employed the first six months, and £900 the second six months; they gained £1200. Required each person's share of the gain.

$$\begin{array}{rcl} 500 \times 4 = 2000 \\ 800 \times 8 = 6400 \end{array}$$

8400 A's money and time.

$$\begin{array}{rcl} 400 \times 6 = 2400 \\ 900 \times 6 = 5400 \end{array}$$

7800 B's money and time.

8400

16200 Sum of the products.

£.	£.	s.	d.	
As 16200 : 1200 :: 8400	: 622	4	5 $\frac{1}{4}$	54 A's share.

As 16200 : 1200 :: 7800	: 577	15	6 $\frac{1}{2}$	108 B's share.
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3. Three merchants, A, B, and C, entered into partnership, A had £600 employed in the business 12 months, B £700 for 10 months, and C £500 for 8 months; they gained £1200. Required each person's share.

Or thus :

On the First of January. A commenced business with a capital of £600, on the 1st of March following, he took in B with £700, two months after this, they took in C, with £500, by the end of the year, they gained £1200. Required each person's share of the same.

Ans. A's £474 14s. 6d. 48 r. B's £461 10s. 9d. 168 r. and C's £263 14s. 8 $\frac{1}{2}$ d. 148. r.

NOTE.—Here, from the First of January to the end of the year is 12 months, the time A was in trade; from the First of March to the end of the year, is 10 months, the time B was in trade, and two months after the First of March to the end of the year is eight months C was in the trade.

4. Two merchants, A and B, entered into partnership, on the the First of January. A brought into the business £500, and on the First of March, £300 more; two months after this, he took out £200, and on the First of September, he brought in £400. B brought into business £700, and on the First of April, took out £100, and on the First of October, he brought in £600. At the end of the year they find they have gained £1500. Required each person's share of the same.

500 $\times$ 2 = 1000 Product of A's money and time for two
300 months, being from the 1st of January
— to the First of March.

800 $\times$ 2 = 1600 Ditto, from the 1st of March to the 1st
200 of May.

600 $\times$ 4 = 2400 Ditto, from the 1st of May to the 1st
400 of September.

1000 $\times$ 4 = 4000 Ditto, from the 1st of September to the
— end of the year.

9000 Product of A's money and time.

700 $\times$ 3 = 2100 Product of B's from 1st Jan. to 1st April.
100

600 $\times$ 6 = 3600 Ditto from 1st April to 1st Oct.
600

1200 $\times$ 3 = 3600 Ditto from 1st Oct. to the end.

9300 Product of B's money and time.
9000

18300 Sum of the products.

	£.		£.	s.	d.	
As 18300 : 1500 :: 9000 :		737	14	1		13200 A's share.
As 18300 : 1500 :: 9300 :		762	5	10 $\frac{3}{4}$		5100 B's share.

5. Two persons, A and B, rent a field, for which they pay £20; A had 12 oxen on it 4 months, and B, 9 oxen for 5 months. Required each person's share of the rent.

Ans. £10 6s. 5½d. 63 A's share, £9 13s. 6½d. 30 B's share.

6. Three persons, A, B, and C, rent a field for which they have to pay 20 guineas; now, A pastured 30 sheep for 4 months, B, 40 sheep for 5 months, and C, 25 sheep for 6 months. Required each person's share of the rent.

Ans. £5 7s. 2½d. 11 A's share, £8 18s. 8½d. 34 B's share, £6 14s. 0½d. 2 C's share.

7. Three merchants, A, B, and C, entered into partnership, A had £500 employed in the business 9 months, B, £700, 6 months, and C with £800, 5 months; they gained £1200. Required each person's share of the gain.

Ans. £425 3s. 11d. 124 A's share, £396 17s. 0d, 48 B's share, £377 19s. 0½d. 82 C's share.

8. On the 1st of April, A began trade with £600, on the 1st of July, he took in B with a capital of £800, on the 1st of August, they took in C, with a capital of £700; by the end of the year they found, since A commenced business, a clear gain of £1500. Required each person's share of the same.

Ans. £591 4s. 9½d. 33 rem. A's share, £525 10s. 11½d. 75 rem. B's share, £383 4s. 2¾d. 29 rem. C's share.

9. Three merchants join in company for 18 months; D puts in £500, and at 5 months end takes out £100, at 10 months end puts in £400, and at the end of 14 months takes out £200; E puts in £600, and at the end of 4 months £200 more, at 12 months he takes out £100, but puts in £300 at the end of 15 months: F puts in £800, and at 6 months takes out £300, at the end of 12 months puts in £600, but takes out that and £100 more at the end of 15 months. They gained £2000. Required each man's share of the gain.

Ans. D, £556 9s. 5½d. 31800 rem.

E, £765 16s. 9½d. 22200 rem.

F, £677 13s. 8½d. 18600 rem.

INVOLUTION.

INVOLUTION is the raising of any number or quantity to a given power, by multiplication. A power of a given number is the product arising from multiplying it by itself the required number of times.

Thus, let 5 be the given number to be raised, which is called the first power or root.

Then $5^2 = 5 \times 5 = 25$, the square, or 2nd power of 5.

$5^3 = 5 \times 5 \times 5 = 125$ the cube, or 3rd power of 5.

$5^4 = 5 \times 5 \times 5 \times 5 = 625$, the biquadrate, or 4th power of 5.

$5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 78125$, the 7th power of 5.

NOTE.—The small figures, 2, 3, 4, and 7, placed over the fives, are called the indices, and denote the power to which the number or quantity is to be raised.

RULE.—Multiply the given number by itself, for the square; twice by itself, for the cube; three times by itself, for the biquadrate, or fourth power, and so on.

When any high power is required, multiply two or more of such inferior powers as will make it together, and their product will be the power required.

EXAMPLES.

1. Required the 8th power of 5.

5	Or thus,	5
5		5
—		—
25 the square.		25 = 2nd power.
5		25 = ditto.
—		—
125 the 3rd power.		125
5		50
—		—
625 the 4th ditto.		625 = 4th ditto.
5		625 = 4th ditto.
—		—
3125 the 5th ditto		390625 = 8th ditto.
125 = 3rd ditto.		
—		
390625 = 8th ditto.		

2. Required the 4th power of $\frac{2}{3}$.

Here $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$ Ans.

3. Required the square or 2nd power of 25. *Ans.* 625.
 4. Required the cube or 3rd power of 56. *Ans.* 175616.
 5. Required the 4th power of 57. *Ans.* 10556001.
 6. Required the 5th power of 1.05. *Ans.* 1.2762815625.
 7. What number is equal to the 9th power of 5 ?
Ans. 1953125.
 8. What number has 2.5 for its biquadrate or 4th root ?
Ans. 39.0625.
 9. What vulgar fraction is equal to the 4th power of .04 ?
Ans. $\frac{1}{390625}$
 10. Express the 5th power of $\frac{1}{3}$ in decimals? *Ans.* .004115222

EVOLUTION OR THE EXTRACTION OF ROOTS.

EVOLUTION is the reverse of INVOLUTION ; being the method of finding the square root, the cube root, the biquadrate root, &c. of any given number.

In INVOLUTION, there is no number of which we cannot find the exact power, while in EVOLUTION or the extraction of roots, there are many numbers which do not admit of an exact root.

The exact root is called a rational root.

These roots which approximate nearer and nearer to the exact root, but never arrive at it, are called surd roots.

Thus the square root of 4 is rational, being 2 ; but the square root of 2 is a surd root ; also the cube root of 8 is a rational root, being 2 ; but the cube root of 9 is a surd root, or such as cannot be exactly found.

The root of a quantity is sometimes denoted by writing the character $\sqrt{\quad}$, before it, with the index of the root in it, or by placing a fractional index over it :

Thus $\sqrt{7}$ or $7^{\frac{1}{2}}$ denotes the square root of 7.

$\sqrt[3]{7}$ or $7^{\frac{1}{3}}$ ditto cube root of 7.

$\sqrt[4]{7}$ or $7^{\frac{1}{4}}$ ditto biquadrate root of 7.

$\sqrt[n]{7}$ or $7^{\frac{1}{n}}$ ditto nth. root of 7.

SQUARE ROOT,

CASE I.—*To extract the square root of any whole number or decimal fraction.*

RULE 1.—Divide the given number into periods of two figures each, beginning at the unit's figure.

2. In whole numbers, put a dot or point over the unit's figure, one over the hundreds, and so on, over every second figure, from right to left.

3. In decimals, put a dot or point over the hundreds, one over the tens of thousands, and so on, from left to right. If the decimal be composed of an odd number of figures, a cipher may be placed to the right of it, to complete the period.

4. Find the greatest square number contained in the first or left hand period, and put its root for a quotient. Subtract this square number from the first period, and to the remainder bring down the next period for a dividend.

5. Double the quotient figure, for the first portion of the divisor; then find what figure must be placed to the right of it, so that the result being multiplied by this figure, the product may be equal to, or the nearest less number to the dividend. This figure, thus annexed to the divisor, must be placed in the quotient.

6. Subtract this product from the dividend, and bring down the next period to the remainder for a new dividend. Double the figure on the right, in the last divisor, and find what figure must be annexed to the right of this quantity, so that the result being multiplied by this figure the product may be equal to, or the nearest less number to the new dividend. This figure, last annexed in the divisor, must be placed in the quotient or root. Proceed thus till all the periods be brought down, and the quotient will be the square root required.

7. If there be no remainder, the root is called rational; but if there be a remainder. double ciphers must be annexed to it, and the operation proceeded with, as above, to any number of places required. The figures thus found are decimals, and the root is called an irrational or surd root.

NOTE.—If the square root of a repetend be required, the repeating figures must be added to the remainder instead of ciphers. Proof by Multiplication see examples 1st and 2nd.

EXAMPLES.

1. Required the square root of 106929.

106929 (327 root required.

$$\begin{array}{r} 9 \\ \hline 62 \overline{)169} \\ \underline{124} \end{array}$$

$$\begin{array}{r} 647 \overline{)4529} \\ \underline{4529} \end{array}$$

327

327

2289

654

981

106929 Proof.

2. Required the square root of 246.

246 (15.684 + Ans.

1

$$\begin{array}{r} 25 \overline{)146} \\ \underline{125} \end{array}$$

$$\begin{array}{r} 306 \overline{)2100} \\ \underline{1836} \end{array}$$

$$\begin{array}{r} 3128 \overline{)26400} \\ \underline{25024} \end{array}$$

$$\begin{array}{r} 31364 \overline{)137600} \\ \underline{125456} \end{array}$$

12144

15.684

15.684

62736

125472

94104

78420

15684

245,987856

12144

246.000000 Proof.

3. Required the square root of 5678.243.

NOTE — Here the decimal part contains an odd number of figures ; therefore, annex a cipher, in order to complete the period in the decimal part, as follows :—

5678,2430

Ans. 75.354 +

4. Required the square root of 625.

Ans. 25.

5. Required the square root of 70225.

Ans. 265

6. Required the square root of 194.

Ans. 13.9283882 +

7. Required the square root of 2.

Ans. 1.4142135 +

8. Required the square root of 998001.

Ans. 999.

9. Find the square root of 178857.

Ans. 422.914.

10. Find the square root of 75.347.

Ans. 8.68026.

11. The square of a certain number is 944784 ; required the number.

Ans. 972.

12. What number is that whose square is 788544 ?

Ans. 888.

13. A square field contains 106929 square yards. What the length of its side in feet ?

Ans. 981 feet.

14. If the area of a circle be 729 square yards ; required the side of a square, of equal area in feet. *Ans.* 81

NOTE.—The following rule for extracting the square root in long calculations, will be found useful.

RULE.—Find the square root by the above rule, to one figure more in the root than half the number of figures required. Take the next divisor for a common divisor to the remainder, annexing as many ciphers as may be required to complete the division, for the remaining part of the root see the following Example.

Required the square root of 5, to 10 places of decimals.

$$\begin{array}{r}
 5(2.23606 \\
 \quad 4 \\
 \hline
 42)100 \\
 \quad 84 \\
 \hline
 443)1600 \\
 \quad 1329 \\
 \hline
 4466)27100 \\
 \quad 26796 \\
 \hline
 447206)3040000 \\
 \quad 2683236 \\
 \hline
 447212)3567640(79775 \\
 \quad 3130484 \\
 \hline
 \quad 4371560 \\
 \quad 4024908 \\
 \hline
 \quad 3466520 \\
 \quad 3130484 \\
 \hline
 \quad 3360360 \\
 \quad 3130484 \\
 \hline
 \quad 2298760 \\
 \quad 2236060 \\
 \hline
 \quad 62700
 \end{array}$$

Therefore, the root is 2.2360679775.

CASE II.—*To extract the square root of a vulgar fraction.*

RULE.—The square root of a vulgar fraction may be found by taking that root of the numerator for a new numerator, and the root of the denominator for a new denominator. But this rule cannot be strictly followed with advantage: for when the root of a certain fraction is required, it frequently happens that it may be reduced either to lower or higher terms with advantage, or in many cases, it is advantageous to reduce the fraction to a decimal before we extract the root: care being taken to have an even number of decimal places.

EXAMPLES.

1. Required the square root of $\frac{392}{112}$.

Here $\frac{392}{112} = \frac{49}{14}$; then $\sqrt{\frac{49}{14}}$ or $(\frac{49}{14})^{\frac{1}{2}} = \frac{7}{\sqrt{14}} = .875$ Ans.

Or thus:

$\frac{392}{112} = .765625$; then $\sqrt{.765625} = .875$. Ans.

2. Required the square root of $\frac{7}{11\frac{1}{2}}$

Here $\frac{7}{11\frac{1}{2}} = \frac{49}{88}$; then $\sqrt{\frac{49}{88}}$ or $(\frac{49}{88})^{\frac{1}{2}} = \frac{7}{\sqrt{88}}$ Ans.

3. Required the square root of $\frac{728}{88}$ Ans. $\frac{1}{2}$

4. Required the square root of $\frac{9}{16}$ Ans. .94868

5. Required the square root of $15\frac{1}{2}$ Ans. 3.9157 +

6. Required the square root of $\frac{449}{567}$ Ans. .8

7. Required the square root of $\frac{275}{339}$ Ans. .71428 +

8. Required the square root of $\frac{29}{54}$ Ans. .531816 +

9. Required the square root of $\frac{54}{16}$ Ans. $\frac{1}{2}$ or .583

10. Required the square root of $\frac{6\frac{1}{2}}{4\frac{1}{2}}$ of $\frac{9}{12}$ Ans. .625

11. Find the square root of .34 of 1.75. Ans. .77136

12. Required the square root of $\frac{2}{3}$ of $\frac{3}{4}$ of .7854. Ans. .56049

13. Express the square root of $\frac{4}{5}$ of $\frac{6}{7}$ of 3.571428. Ans. 1.428571

CUBE ROOT.

THE CUBE ROOT of a number is such a quantity, as when raised to its third power, is equal to the given number. Thus, the cube root of 8 is 2; for $2 \times 2 \times 2 = 8$. The cube root of 1728 is 12; for $12 \times 12 \times 12 = 1728$.

Cubes...	1	8	27	64	125	216	343	512	729
Roots ...	1	2	3	4	5	6	7	8	9

To find the cube root of any given number.

RULE.—Divide the given number into periods of three figures each, beginning at the units figure; as in the square root. If the number contain a decimal, the period must be made complete by annexing ciphers to the right hand of it.

The cube of your first period take,
 And of its root a quotient make;
 Which root into a cube must grow,
 And from your period taken fro'.
 To the remainder, then you must
 Bring down another period just,
 Which being down, then you must see,
 Your numbers straight divided be,
 By just three hundred times the square
 Of what your quotient figures are;
 Which do so that you in may take,
 The fact your quotient figures make.
 Last squar'd and multipl'd by th' rest,
 And product thirty times express'd.
 The cube of your last found figure too,
 You must put in, if right you do.
 Repeat the work, and so descend
 From point to point unto the end;
 That done, if ought remain, you shall
 Add triple ciphers for a decimal.

NOTE.—In extracting the cube root of a decimal, care must be taken that the decimal places be three, or some multiple of three; because there are three times as many decimal places in the cube as in the root.

EXAMPLES.

1. Required the cube root of 12895213625.

$$\begin{array}{r} 12895213625(2345 \text{ Ans.} \\ 2^3 = \quad \quad \quad 8 \end{array}$$

$$2^2 \times 300 = 1200)4895$$

$$\underline{3600}$$

$$540 = 3^2 \times 2 \times 30$$

$$27 = 3^3$$

$$\underline{4167}$$

$$(23)^2 \times 300 = 158700)728213$$

$$\underline{634800}$$

$$11040 = 4^2 \times 23 \times 30$$

$$64 = 4^3$$

$$\underline{645904}$$

$$(234)^2 \times 300 = 16426800)82309625$$

$$\underline{82134000}$$

$$175500 = 5^2 \times 234 \times 30$$

$$125 = 5^3$$

$$\underline{82309625}$$

Proof,

$$(2345)^3 = 2345 \times 2345 \times 2345 = 12895213625.$$

NOTE.—Here the root is rational. When there is a remainder the root is irrational or a surd root.

The following rule may be used with advantage, particularly in long calculations.

1. Find the cube root by the common method, to one figure more in the root than half the figures required.

2. Divide the remainder with ciphers annexed, by three times the square of the figures in the root already found, for the remaining part of the root, see Example.

Find the cube root of 598 to 7 places of figures.

$$\begin{array}{r}
 598(8.424 \\
 512 \\
 \hline
 19200)86000 \\
 \hline
 76800 \\
 3840 \\
 64 \\
 \hline
 80704 \\
 \hline
 2116800)5296000 \\
 \hline
 4233600 \\
 10080 \\
 8 \\
 \hline
 4243688 \\
 \hline
 212689200)1052312000 \\
 \hline
 850756800 \\
 404160 \\
 64 \\
 \hline
 851161024 \\
 \hline
 (8424)^2 \times 3 = 212891328)2011509760(944 \\
 1916021952 \\
 \hline
 954878080 \\
 851565312 \\
 \hline
 1033127680 \\
 851565312 \\
 \hline
 181562368 \\
 \hline
 \end{array}$$

Hence 8.424944 the cube root, correct to 7 places of figures.

EXAMPLES FOR PRACTICE.

1. Required the cube root of 1728. *Ans.* 12.
2. Required the cube root of 1860867. *Ans.* 123.
3. Required the cube root of 14348907. *Ans.* 243.
4. Required the cube root of 48627125. *Ans.* 365.
5. Required the cube root of 486217.25* *Ans.* 78.634.
6. Required the cube root of 598. *Ans.* 8.42494.
7. Required the cube root of 789. *Ans.* 9.24043.
8. Required the cube root of $\frac{27}{64}$. *Ans.* $\frac{3}{4}$.
9. Required the cube root of $3\frac{3}{8}$. *Ans.* $\frac{7}{2}$.
10. Required the cube root of $3\frac{3}{8} \div 15\frac{1}{2}$. *Ans.* $\frac{3}{5} = .6$
11. Required the cube root of 23. *Ans.* 2.843807.
12. Required the cube root of 154. *Ans.* 5.360108.
13. Required the cube root of $71\frac{1}{2}$. *Ans.* 4.14324638.
14. Express the cube root of 0.25 of 3.345. *Ans.* .942175.

* In question 5, a cipher must be annexed to the right of the decimal, in order to make a complete period.

EXTRACTION OF ROOTS IN GENERAL.

The extraction of roots is more readily performed by the use of logarithms, as follows:—Find the logarithm of the given number; and divide it by 2, when the square root is required; by 3, when the cube root is required; by 4, when the fourth root is required, and so on. The natural number corresponding to this quotient or logarithm, will be the root required.

EXAMPLES.

1. Required the square, the cube, and the biquadrate roots of 7854.

Here, the logarithm of 7854 is 3.8950909;

Then $3.8950909 \div 2 = 1.9475454$, and the number corresponding to this log. is 88.622, the square root.

3)3.8950909

1.2983636, the corresponding number is 19.877 c. root.

4)3.8950909

.9737727, the corresponding No. 9.4139, fourth root.

2. Required the 5th, 6th, and 7th roots of 94678.

Ans. 5th root, 9.8912; 6th, 6.7511, and 7th, 5.1391.

ARITHMETICAL PROPORTION.

ARITHMETICAL PROPORTION is the relation which quantities of the same kind have to each other with respect to their differences.

Three quantities are in arithmetical proportion, when the difference between the 1st and 2nd is equal to the difference between the 2nd and 3rd; as 4, 6, and 8.

Four quantities are in arithmetical proportion when the difference between the 1st and 2nd is the same as the difference between the 3rd and 4th; thus 2, 6, 12, and 16 are arithmetical proportionals.

When three numbers are in arithmetical proportion, the sum of the 1st and 3rd is equal to twice the 2nd.

When four numbers are in arithmetical proportion, the sum of the 1st and 4th is equal to the sum of the 2nd and 3rd.

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION denotes a series of numbers or quantities of the same kind, having the same common difference:—

Thus, 2, 5, 8, 11, 14, 17, &c. (A), is an increasing series in arithmetical progression, having 3 for the common difference.

Again, 18, 16, 14, 12, 10, 8, 6, 4, 2, (B), is a decreasing series in arithmetical progression, having 2 for the common difference. The first and last terms of the series are called the extremes. Some of the properties of these series are as follows:—

1. The sum of any two terms in the series is equal to the sum of any other two equally distant from them:

$$\text{In Series (A.) } 2 + 17 = 5 + 14 = 8 + 11 = 19.$$

2. If there be a middle term, the sum of any two terms equally distant from it will be equal to twice this term.

In series (B.)

$$18 + 2 = 16 + 4 = 14 + 6 = 12 + 8 = 10 \times 2 = 20.$$

3. The sum of any series of numbers in arithmetical progression is equal to the sum of the extremes, multiplied by half the number of terms.

$$\text{Thus } (18+2) \times 4\frac{1}{2} = 20 \times 4\frac{1}{2} = 90, \text{ the sum of series (B.)}$$

4. The difference between the extremes is equal to the number of terms, less one, multiplied by the common difference.

Thus, in series A, $(6-1) \times 3 = 5 \times 3 = 15 = (17-2)$

... B, $(9-1) \times 2 = 8 \times 2 = 16 = (18-2)$

5. If out of any series of numbers in arithmetical progression, any number of equidistant terms be taken, these numbers will form a series of numbers in arithmetical progression.

Thus, 2, 5, 8, 11, 14, 17;

Then will 5, 11, 17 be a series of numbers in arithmetical progression.

Also 18, 16, 14, 12, 10, 8, 6, 4, 2;

Then will 16, 12, 8, 4, be in arithmet. pro.

PROBLEM I.—*Given the first and last terms, that is, the extremes, and the number of terms of a series of numbers in arithmetical progression to find their sum.*

RULE.—Multiply the sum of the extremes by half the number of terms, and the product will be the sum of the series.

EXAMPLES.

1. What is the sum of 2, 3, 4, 5, 6, 7, and 8?

Here $(2 + 8) \times 3\frac{1}{2} = 10 \times 3\frac{1}{2} = 35$. *Ans.*

2. What is the sum of $9\frac{1}{2}$, $7\frac{1}{2}$, $5\frac{1}{2}$, and $3\frac{1}{2}$? *Ans.* 26.

3. How many strokes does the hammer of a clock strike in 12 hours? *Ans.* 78.

4. If 100 stones be placed in a straight line, one yard asunder, and the first one yard from a basket; how many miles will a person have to travel, in order to gather them up singly, returning with each stone to the basket?

Ans. 5 miles 1300 yards.

5. If 300 stones be placed in a straight line, two yards asunder, and the first two yards from a basket; how far will a person have to travel, in order to gather them up singly, returning with each stone to the basket?

Ans. 102 miles, 1080 yards.

PROBLEM II.—*Given the extremes and the arithmetical ratio, or common difference, to find the number of terms.*

RULE.—Divide the difference of the extremes by the arithmetical ratio or common difference, and this quotient increased by one, will be the number of terms.

EXAMPLES.

1. The first and last terms of a series of numbers in arithmetical progression are 2 and 17, and the common difference 3; required the number of terms.

Here $(17 - 2) \div 3 = 15 \div 3 = 5$; then $5 + 1 = 6$ *Ans.*

2. The first and last terms of a series of numbers in arithmetical progression are 18 and 2, and the arithmetical ratio or common difference 2; required the number of terms.

Ans. 9.

3. The first and last terms are $2\frac{1}{2}$ and 20, and the common difference $\frac{1}{2}$; required the number of terms.

Ans. 36.

4. A person going a journey, travels 3 miles the first day, 7 the second, 11 the third, &c., and on the last day 47 miles. How many days did he travel, and how many miles in all?

Ans. 12 days, 300 miles.

PROBLEM III.—*Given the extremes of a series of numbers in arithmetical progression, and the number of terms, to find their common difference.*

RULE.—Divide the difference of the extremes, by the number of terms less one, and the quotient will be the common difference.

EXAMPLES.

1. The first and last terms of the series are 2 and 17, and the number of terms 6. Required the common difference.

Here $(17 - 2) \div (6 - 1) = 15 \div 5 = 3$ *Ans.*

2. The first and last terms are $2\frac{1}{2}$ and 20, and the number of terms 36. Required the common difference and sum of the series.

Ans. $\frac{1}{2}$ common difference, 405 sum.

3. A person having to travel a certain number of miles in 12 days; finds he can, by travelling 3 miles the first day, and making a regular daily increase till he travels 47 on the last day, perform the journey. Required his increase per day, and the distance he travelled.

Ans. 4 miles daily increase, 300 miles distance.

PROBLEM IV.—*Given the number of terms, their sum, and common difference, to find the least term.*

RULE.—Divide the sum of the series by the number of terms; from this quotient, subtract half the product of the number of terms less one by the common difference, and the remainder will be the least term.

EXAMPLES.

1. The number of terms is 10, their sum 155, and common difference 3. Required the least term of the series.

Here $155 \div 10 = 15\frac{1}{2}$; and $(10 - 1) \times 3 \div 2 = 9 \times 3 \div 2 = 27 \div 2 = 13\frac{1}{2}$; then $15\frac{1}{2} - 13\frac{1}{2} = 2$ least term.

2. The number of terms is 36, their sum 405, and the common difference $\frac{1}{2}$. Required the least term of the series.

Ans. $2\frac{1}{2}$.

3. A man travels 10100 miles in 100 days; the first day he travels a certain number of miles, and each succeeding day two miles more. How many miles did he travel the first day?

Ans. 2 miles.

4. A young lady is to receive £558 2s. 11d. in one year, for her portion, by successive daily payments, each succeeding payment to exceed the former by 2d. What will her first payment be?

Ans. 3d.

PROBLEM V.—Given the least term, the common difference, and the number of terms to find the greatest term.

RULE.—Multiply the number of terms by the common difference, to this product add the least term; from this sum subtract the common difference, and the remainder will be the greatest term.

EXAMPLES.

1. The least term is 3, the common difference 2, and the number of terms 365. Required the greatest term.

Here $(365 \times 2) + 3 = 730 + 3 = 733$; then $733 - 2 = 731$ *Ans.*

2. The least term is 4, the common difference 4, and the number of terms 300. Required the greatest term.

Ans. 1200.

4. Bought 100 yards of cloth, for the first yard I paid 3d. for the second 7d, and so on, for each succeeding yard. Required the price of the last yard, and the value of the whole. *Ans.* £1. 13s. 3d. last yard, and £83 15s. for the whole.

PROBLEM VI.—*Given the number of terms, their common difference, and the greatest term, to find the least term.*

RULE.—Multiply the number of terms less 1, by the common difference; and subtract this product from the greatest term, and the remainder will be the least term.

EXAMPLES.

1. The greatest term in a series of numbers in arithmetical progression is 23, the number of terms 8, and the common difference 3. Required the least term.

Here $(8 - 1) \times 3 = 7 \times 3 = 21$; and $23 - 21 = 2$ *Ans.*

2. The greatest term is 20, the number of terms 36, and the common difference $\frac{1}{2}$. Required the least term. *Ans.* $2\frac{1}{2}$.

3. A person who had a daily allowance for one year, each day's pay being 2d. more than the preceding, received £3. 11d. for the last day. How much was paid for the first day, and what sum for the year? *Ans.* 3d. for the first day, £558 2s. 11d. for the year.

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is a series of numbers or quantities which increase or decrease by a common ratio, as 2, 4, 8, 16, 32, 64, &c., and 81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c., are series in geometrical progression; the ratio of the first being 2, and that of the second $\frac{1}{3}$. The first is called an increasing series, and the second a decreasing series.

The first and last terms are called the extremes, and the common multiplier or divisor the ratio.

The middle term is called the mean, when the numbers are odd; but when even, the two middle terms are called means.

If three numbers be in geometrical progression the product of the extremes is equal to the square of the mean.

Thus, if the geometrical series 4, 8, 16, be taken ; $4 \times 16 = 8 \times 8$.

If four numbers, in geometrical progression, be taken, as 4, 8, 16, 32 ; Then $4 \times 32 = 8 \times 16$.

If the series 2, 4, 8, 16, 32, 64, 128 be taken ;

Then $2 \times 128 = 16 \times 16$.

If the series 81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$ be taken ;

Then $81 \times \frac{1}{27} = 3 \times 1$;

Or, the product of the two extremes will be equal to the product of any two means, equidistant from the extremes.

Then $2 \times 128 = 4 \times 64$; Or $81 \times \frac{1}{27} = 9 \times \frac{1}{3}$.

The equidistant terms, of a series of numbers in geometrical progression, are in geometrical progression.

Thus, in the series 2, 4, 8, 16, 32, 64, 128, &c., whose ratio is 2.

Then 4, 16, 64, and the equidistant terms, are in geometrical progression, whose ratio is 4.

PROBLEM I.—*Given the first term, the ratio, and the number of terms to find the last or any other assigned term.*

RULE—Find such a power of the ratio as is denoted by the number of terms less one, multiply this result by the first term, and the product will be the last term.

NOTE.—If the first term and ratio be the same, find such a power of the ratio as is denoted by the number of terms, and the result will be the last term.

EXAMPLES.

1. The first term of a series of numbers in geometrical progression is 2, the ratio 3, and the number of terms 12. Required the last term.

Here $12 - 1 = 11$ the number of terms less one ;

Then the ratio 3 raised to the 11th. power as follows :

Here $3^{11} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\times 3 \times 3 = 177147$ the 11th. power of the ratio.

Then $177147 \times 2 = 354294$ the last term.

2. The first term is 5, the ratio 3, and the number of terms 17. Required the last term of the series. *Ans.* 215233605.

3. The first term of a decreasing series in geometrical progression is 12, the ratio $\frac{1}{2}$ or divisor 2, and the number of terms 9. Required the last term. *Ans.* $\frac{3}{64}$.

4. Bought 25 yards of cloth, the first yard cost 2d., the second 4d., the third 8d., &c. What was the value of the last yard? *Ans.* £139810 2s. 8d.

5. A thrasher worked 30 days, for the first day's work he received 4 wheat corns, for the second 12, and for the third, 36, &c. What did he receive for his last day's work; if 8000 grains of wheat make a pint, and the wheat be worth 8s. per bushel? *Ans.* £214469929 5s. 3½d.

6. A person wishes to dispose of a horse for the value of the last nail in his shoes, on the following terms, viz. one farthing for the first nail, two farthings for the second, one penny for the third, and so on; admitting the 4 shoes to contain 32 nails. What would the horse be sold for?

Ans. £2236962 2s. 8d.

PROBLEM II.—*Given the extremes and the ratio of a series of numbers in geometrical progression, to find their sum.*

RULE.—Multiply the greatest term of the series by the ratio, and divide the difference of this product and the least term, by the difference between one and the ratio, and the quotient will be the sum required.

EXAMPLES.

1. The first term is 2, the last term 486, and the ratio 3. Required the sum of the series.

Here $(486 \times 3) - 2 = 1458 - 2 = 1456$, and $3 - 1 = 2$; then $1456 \div 2 = 728$ *Ans.*

2. The first term of a decreasing geometrical series is 162, the last term $\frac{3}{2}$, and the ratio $\frac{1}{3}$. Required the sum.

Here $162 \times \frac{1}{3} - \frac{3}{2} = 54 - \frac{3}{2} = 48\frac{1}{2}$; and $\frac{1}{3} - 1 = -\frac{2}{3}$; then $48\frac{1}{2} \div -\frac{2}{3} = -72\frac{3}{4}$ *Ans.*

3. The first term is 3, the last term 6144, and the ratio 2. What is the sum of the series? *Ans.* 12286.

4. The first term is $1\frac{1}{2}$, the last term $29524\frac{1}{2}$, and the ratio 3. What is the sum of the series? *Ans.* 44286.

5. The first term is 100, the last term $12\frac{1}{2}$, and the ratio $\frac{1}{2}$. What is the sum of the series? *Ans.* $199\frac{1}{2}$.

6. Required the value of the 25 yards of cloth, mentioned in the 4th. question, problem 1st. *Ans.* £279620 5s. 2d.

7. How much money would the thrasher, in the fifth question, problem first, receive for his 30 days' wages?

Ans. £321704893 17s. 11½d. nearly.

8. What would be the value of a horse on the following terms, viz. one farthing being paid for the first nail in the shoe, 2 farthings for the second, a penny for the third, and so on; admitting the 4 shoes to contain 9 nails each?

Ans. £71582788 5s. 3½d.

9. One Sessa, an Indian, having first discovered the game of chess, showed it to his prince, Shehram, who was so delighted with the invention, that he bid him ask what he would, as a reward for his ingenuity: upon which, Sessa requested that he might be allowed one grain of wheat for the first square on the chess-board, two for the second, four for the third, and so on, doubling continually to 64, the whole number of squares: now, supposing a pint to contain 8000 grains; it is required to find what number of ships, each carrying 1000 tons, might be freighted with the produce, allowing 40 bushels to a ton?

Ans. 900719925.474099199951171875 ships.

ANNUITIES.

The term Annuity is understood to signify any interest of money, rent, or pension, payable from time to time, at stated periods; and these payment may take place yearly, half-yearly, quarterly, &c,

Annuities are of two kinds, namely, possession and reversion. Annuities in possession are such as are entered upon, and annuities in reversion are such as cannot be entered upon, till some future period or event takes place.

An annuity is in arrears, when it is not paid for any certain period after it becomes due; and the sum of all the single payments, together with the interest due thereon, is the amount.

N.B.—When no period of payment is stated, yearly payments are understood.

When an annuitant disposes of his annuity, the sum paid for it is the present worth.

CASE I.—*To find the amount of an annuity, at simple interest.*

RULE.—Find the sum of the series 1, 2, 3, 4, &c., to the number of years, less one; and multiply the interest of the given annuity for one year by this sum, and the product will be the amount of interest due.

Multiply one year's annuity by the number of years, to which product add the whole interest, and the sum will be the amount required.

NOTE.—This rule is obvious, for whatever the time may be, there is due upon the first year's annuity, as many years' interest as the whole number of years, less one; and one less upon every succeeding year's annuity.

EXAMPLES.

1. What is the amount of an annuity of £360, nine years in arrears, at 5 per cent., per annum, simple interest?

Here $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.

5 is $\frac{1}{20}$ (360

18 one year's interest on £360.

36

648 interest due.

$360 \times 9 = 3240 = 9 \text{ years' annuity.}$

£3888 amount required.

2. Required the amount of an annuity of £75, seven years in arrears, at 4 per cent. per annum. *Ans.* £588.

3. What is the value of an annuity of £150, twelve years in arrears, at $4\frac{1}{2}$ per cent. per annum.

Ans. £2245 10s. 0d.

4. What is the amount of a yearly annuity of £300, six years in arrears, payable half yearly, at 5 per cent. per annum? *Ans.* £2047 10s.

CASE II.—*To find the present worth of an annuity, at simple interest.*

RULE.—Find the present worth of each yearly payment separately, by the rule for discount; and their sum will be the present worth required.

NOTE.—The present worth or value of an annuity, is that sum which, being improved at interest for the given time, becomes equal to the amount of the annuity.

EXAMPLES.

1. What is the present worth of an annuity of £50, to continue 4 years, at 5 per cent. per annum, simple interest?

£.	£.	£.	£.	s.	d.	
As 105	: 100	:: 50	: 47 12 4½			present worth of the 1st year.
110	: 100	:: 50	: 45 9 1			ditto 2nd.
115	: 100	:: 50	: 43 9 6½			ditto 3rd.
120	: 100	:: 50	: 41 13 4			ditto 4th.

178 4 4½ Ans.

2. Required the present worth of an annuity of £150, to continue 5 years, at 4 per cent. per annum.

Ans. £671 7s. 1½d.

CASE III.—To find the amount of an annuity, at compound interest.

RULE.—Find the sum of as many terms of a geometrical series, as there are payments in the annuity, having 1 for the first term, and the amount of one pound for the ratio; and the result will be the amount of £1 annuity at compound interest, for the given time.

Multiply this result by the given annuity, and the product will be the amount required.

TABLE.

The amount of £1 for 1 year,		} $1 + \frac{5}{100} = \text{£}1.05.$
at 5 per cent. per annum, is		
at $4\frac{1}{2}$	ditto	$1 + \frac{4\frac{1}{2}}{100} = \text{£}1.0475.$
at $4\frac{1}{4}$	ditto	$1 + \frac{4\frac{1}{4}}{100} = \text{£}1.045.$
at $4\frac{1}{8}$	ditto	$1 + \frac{4\frac{1}{8}}{100} = \text{£}1.0425.$
at 4	ditto	$1 + \frac{4}{100} = \text{£}1.04.$
at $3\frac{1}{2}$	ditto	$1 + \frac{3\frac{1}{2}}{100} = \text{£}1.0375.$
at $2\frac{1}{2}$	ditto	$1 + \frac{2\frac{1}{2}}{100} = \text{£}1.025.$

&c. &c.

EXAMPLES.

1. What is the amount of an annuity of £50, to continue 4 years, at 5 per cent. per annum, compound interest?

Here the series is, $1 + 1.05 + 1.05^2 + 1.05^3 = 4.310125$.

Or thus :

$$\begin{array}{rcl} 1 & = & 1 \\ 1.05 & = & 1.05 \\ 1.05^2 & = & 1.1025 \\ 1.05^3 & = & 1.157625 \end{array}$$

$\underline{\hspace{1.5cm}}$
£4.310125 the amount, £1 annuity for the given time.

Then $4.310125 \times 50 = 215.50625 = \text{£}215 \text{ } 10\text{s. } 1\frac{1}{2}\text{d. } \textit{Ans.}$

2. What is the amount of an annuity of £60, to continue four years, payable half yearly, at 5 per cent. per annum, compound interest?

NOTE.—Here we have 8 separate payments of £30 each ; consequently it may be taken as 8 years, at two and a half per cent. as follows :

$$1 + 1.025 + 1.025^2 + 1.025^3 + 1.025^4 + 1.025^5 + 1.025^6 + 1.025^7 = 8.736114.$$

Then $8.736114 \times 30 = 262.08342 = \text{£}262 \text{ } 1\text{s. } 8\text{d. } \textit{Ans.}$

3. What is the amount of an annuity of £120, to continue 6 years, at 5 per cent. compound interest? *Ans. £816 4s. 7d.*

4. What is the amount of an annuity of £60, to continue 7 years, at $4\frac{1}{2}$ per cent. compound interest?

Ans. £481 2s. 11½d.

5. If a yearly rent of £300, payable half yearly, be forborne 5 years ; to what will it amount at 5 per cent. per annum, compound interest?

Ans. £1680 10s. 1½d.

6. If an annuity of £250 be unpaid for 10 years ; to what will it amount, at $3\frac{1}{2}$ per cent. compound interest?

Ans. £2932 16s. 11½d.

CASE IV.—To find the present worth of an annuity, at compound interest.

RULE.—Divide the amount of £1 annuity for the given time, by such a power of the ratio, of the series, as is denoted by the number of years, and the quotient will be the present worth of £1 annuity.

Multiply this present worth by the annuity, and the product will be the present worth required.

NOTE.—If the annuity be payable half yearly, quarterly, &c., find the amount as per note in Case 3rd.

EXAMPLES.

1. What is the present worth of an annuity of £40, to continue 5 years, at 5 per cent. per annum, compound interest?

$$\begin{aligned} 1. &= 1 \\ 1.05 &= 1.05 \\ 1.05^2 &= 1.1025 \\ 1.05^3 &= 1.257625 \\ 1.05^4 &= 1.21550625 \end{aligned}$$

5.52563125 the amount of £1 annuity,
for 5 years.

Then $1.05^5 = 1.2762815625$, the fifth power of the ratio, and $5.52563125 \div 1.2762815625 = £4.32947$, the present worth of £1 annuity.

Therefore, $4.32947 \times 40 = £173.1788 = £173 \text{ 3s. } 6\frac{3}{4}\text{d.}$, the present worth required.

2. What is the present worth of £100 per annum, to continue 6 years, at 5 per cent. compound interest?

Ans. £507 11s. 4½d.

3. What sum may I give for £320 per annum, to continue 7 years, so as to make 4 per cent. per annum, compound interest?

Ans. £1920 13s. 1¾d.

4. What is the present worth of £400 per annum, payable half yearly, to continue 5 years, at 6 per cent. per annum, compound interest?

Ans. £1706 0s. 9½d.

5. Required the difference between the amount and present worth of an annuity of £500, to continue 4 years, at 5 per cent. per annum, compound interest?

Ans. £382 1s. 9d.

6. Find the different amounts of an annuity of £360, at simple and compound interest, for 8 years, at 5 per cent.

Ans. £3437 13s. 7d., amount at compound interest, and £3384, at simple interest.

CASE V.—*To find the present worth of a freehold estate, or an annuity in perpetuity, that is, for ever, at compound interest.*

RULE.—As the rate per cent. is to £100, so is the yearly rent or annuity to the present worth.

EXAMPLES.

1. What is the worth of a perpetual annuity of £43 13s. per annum, at $4\frac{1}{2}$ per cent.?

As £4 $\frac{1}{2}$: £100 :: £43 13s. : £970 *Ans.*

2. What is the present worth of a freehold estate of £250 per annum, at 5 per cent. per annum. *Ans.* £5000.

CASE VI.—*To find the present worth of annuities, estates, or leases in reversion, at compound interest.*

RULE.—Consider this as an annuity in possession for the time of its continuance, and find its present worth.

Divide this present worth by such a power of the ratio, as is denoted by the number of years between the purchase and possession, and the quotient will be the present worth required.

EXAMPLES.

1. What is the present worth of an annuity of £120, to commence 9 years hence, and continue 11 years, at 6 per cent. per annum, compound interest?

Here $1 + 1.06 + (1.06)^2 + (1.06)^3 + \&c. \text{ to } (1.06)^{10} = 14.971641$

Then $14.971641 \div (1.06)^{11} = 14.971641 \div 1.898298 = 7.886876$, the present worth of £1 annuity for 11 years.

Therefore $7.88687 \times 120 = 946.42512$, the present worth of £120 annuity for 11 years.

Again: $946.42512 \div 1.689478$ (the 9th power of 1.06) = £560.18789 = £560 3s. 9d. the present worth required.

2. The reversion of a freehold estate of £125 per annum, will come into possession 5 years hence; what present money is it worth, allowing the purchaser 5 per cent. per annum, compound interest?

As £5 : £100 :: £125 : £2500, the present worth, if entered to immediately.

Then $(1.05)^5 = 1.2762815625$; and $2500 \div 1.2762815625 = 1958.815416 = £1958 16s. 3\frac{1}{2}d.$ *Ans.*

3. What is the present worth of an annuity of £300, to commence 5 years hence, and continue 7 years, at 5 per cent. per annum, compound interest? *Ans.* £1360 2s. 7 $\frac{1}{2}d.$

DUODECIMALS, OR CROSS MULTIPLICATION.

IN this Rule, every superior place is 12 times its next inferior in the scale of notation:

Thus 12' inches make 1 foot.
 12'' seconds make 1' inch.
 12''' thirds make 1'' second.
 12'''' fourths make 1''' third, &c.

By it, carpenters, joiners, masons, bricklayers, painters, plasterers, plumbers, glaziers, slaters, &c. &c. compute their work.

The dimensions are taken in feet and inches, and the content estimated in square feet and inches.

RULE.—1. Place the dimensions, so that feet may stand under feet, inches under inches, &c.

2. Multiply each term in the multiplicand by the feet in the multiplier, beginning at the lowest term, observing to carry 1 for each 12 in the product to the product of the next superior denomination, placing the remainder of these results under their corresponding terms.

3. Multiply in like manner all the terms of the multiplicand by the inches in the multiplier, and set the result one place more to the right, and so on for any lower denomination; then the sum of these products will be the answer required.

EXAMPLES.

1. Multiply 4 feet 9 inches by 5 feet 7 inches.

<i>feet.</i>	<i>inches.</i>	
4	9	
5	7	
23	9	
2	9	3
26	6	3''

NOTE.—This product is usually read 26 sq. feet 6 in. and 3 parts. Now the 6 is not six square inches, but $\frac{6}{12}$, or $\frac{1}{2}$ of 1 square foot, or 72 square inches; and the 3'' (seconds) denotes $\frac{3}{12}$ of a unit of the 6 inches.

2. Multiply 8 feet 4 inches 9 parts by 7 feet 9 inches 5 parts, or seconds.

<i>feet.</i>	<i>in.</i>	<i>"</i>		
8	4	9		
7	9	5		
58	9	3		
6	3	6	9'''	
	3	5	11	9'''
65	4	3	8	9

NOTE.—Here 9 is $\frac{9}{12}$ of a unit of the 8 ; 8 is $\frac{8}{12}$ of a unit of the 3 ; 3 is $\frac{3}{12}$ of a unit of the 4 ; and 4 is $\frac{4}{12}$ of a square foot, or 48 sq. inches.

3. Multiply 4 feet 6 inches by 3 feet 3 inches.

Ans. 14 feet 7 in. 6 p.

4. Multiply 8 ft. 9 in. by 7 ft. 10 in. *Ans.* 68 ft. 6 in 6''.

5. Multiply 12 ft. 4 in. by 8 ft. 8 in. *Ans.* 106 ft. 10 in. 8 p.

6. Multiply 15 ft. 8 in. by 9 ft. 10 in. *Ans.* 154 ft. 0 in. 8 p.

7. Multiply 15 ft. 3 in. by 12 ft. 5 in. *Ans.* 189 ft. 4 in. 3 p.

8. Multiply 16 ft. 4 in. by 12 ft. 10 in. *Ans.* 209 ft. 7 in. 4 p.

9. Multiply 16 ft. 8 in. 4 p. by 4 ft. 4 in.

Ans. 72 ft. 4 in. 1'' 4'''

10. Multiply 17 ft. 3 in. 6 p. by 9 ft. 8 in.

Ans. 167 ft. 1 in. 10''

11. Multiply 2 ft. 3 in. 6 p. by 8 ft. 4 in. 3 p.

Ans. 19 ft. 1 in. 8'' 10''' 6'''

12. Multiply 6 ft. 9 in. 8 p. by 4 ft. 3 in. 7 p.

Ans. 29 ft. 3 in. 0'' 7''' 8'''

13. Multiply 4 ft. 6 in. 3'' 4''' by 3 ft. 2½ in.

Ans. 14 ft. 6 in. 1'' 8''' 4'''

14. Multiply 3 ft. 6 in. 2'' 3'' by itself.

Ans. 12 ft. 4 in. 3'' 9''' 5''' 0''' 9'''

15. What is the area of a passage floor, its length being 15 feet 6 inches, and breadth 4 feet 9 inches ?

Ans. 73 sq. ft. 7 in. 6 p., or 8 sq. yds. 1 ft. 7 in. 6 p.

16. Required the area of a room floor, its length being 15 ft. 6 in. and breadth 12 ft. 9 in.

Ans. 197 sq. ft. 7 in. 6 p., or 21 sq. yds. 8 sq. ft. 7 in. 6 p.

17. Required the area of a floor, its length being 18 feet 9½ inches, and breadth 15 feet 4 inches.

Ans. 32 sq. yds. 1 in. 8 p.

18. Required the area of a garden wall, its length being 125 feet 6 inches, and the height or breadth 7 feet 3 inches.

Ans. 101 yds. 10 in. 6 p. or 14 roods 27 ft. 10 in. 6 p.

NOTE.—The expense of a floor, wall, &c. may be found, either by Practice or the Rule of Three.

19. A floor is 15 feet 3 inches by 12 feet 9 inches. What will it cost, at 3s. 6d. per square yard?

Here (15 ft. 3 in.) $\times$ (12 ft. 9 in.) = 194 sq. ft. 5 in. 3 p.

As 9 ft. : 3s. 6d. :: 194 ft. 5 in. 3 p.

12	12	12
—	—	—
108	42	2333
12		12
—		—
1296		27999
		42

1296)1175958(907¼d. = £3 15s. 7¼d. *Ans.*

20. Required the expense of the passage floor, mentioned in the 15th question, at 4s. 6d. per square yard.

Ans. £1 16s. 9¼d.

21. The length of slating on a roof is 35 feet 6 inches, and girth 22 feet 3 inches. What will it cost slating, at 2s. 11d. per square yard?

Ans. £12 15s. 11¼d.

22. Required the expense of 8 windows, at 1s. 4d. per square foot, each measuring 4 feet 8 inches by 6 feet 9 inches.

Ans. £16 16s.

23. A roof measures 36 feet 6 inches in length, and 24 feet 6 inches in girth; what will it cost, at £2 15s. per square, of 100 square feet?

Ans. £24 11s. 10d.

24. Required the expense of a wall, which measures 28 feet 9 inches by 12 feet 8 inches, at £1 8s. per square rood of 63 square feet, deducting for 2 doors, each 6 feet 6 inches by 3 feet 2 inches, and 3 windows, each 4 feet by 6 feet 3 inches.

Ans. £5 10s. 2¼d.

25. A ceiling measures 17 feet 6 inches by 15 feet 4 inches. Required the expense of plastering it, at 11¼d. per square yard.

Ans. £1 8s. 6¾d.

26. A room is 17 feet 6 inches long, 15 feet 4 inches broad. What will the walls cost plastering, at 9d. per square yard, the height being 10 feet? *Ans.* £2 14s. 8½d.

27. What will the above room cost papering, with paper at 4d. per yard; deducting for a door 3 feet 6 inches by 6 feet 6 inches, and 2 windows each 4 feet 8 inches by 5 feet 6 inches. A piece of paper measures 12 yards in length, by 21 inches in breadth. *Ans.* £1 16s. 11¾d.

28. A room is 20 feet 6 inches long, 16 feet 2 inches broad, and 10 feet high. What will be the difference in expense between papering and painting it; the paper being 7d. per yard, and painting 10d. per square yard, allowing no deductions for openings? *Ans.* £4 1s. 5¾d. papering, £3 7s. 10½d. painting, 13s. 7d. difference.

29. A window contains 20 panes of glass, each 21½ inches by 16½ inches; what will it cost glazing, at 1s. 8d. per square foot. *Ans.* £4 0s. 4¾d.

30. A roof 28 feet 6 inches by 24 feet 6 inches, is covered with lead, at 7 lb. to the square foot; what will it cost, including workmanship, at £1 10s. per cwt.? *Ans.* £65 9s. 2½d.

CUBIC, OR SOLID MEASURE.

RULE.—Multiply the length, breadth, and thickness continually together, and the product will be the solidity or solid content in the denomination of the dimensions.

EXAMPLES.

1. A piece of timber measures 2 feet 6 inches long, 1 foot 4 inches broad, and 1 foot 2 inches thick. Required its solid content.

Here 2 feet 6 inches = 30 inches, 1 foot 4 inches = 16 in.,
and 1 foot 2 inches = 14 inches.

Then $30 \times 16 \times 14 = 6720$ cubic inches the solidity.

1728) 6720 (3 ft. 10 in. 8 p. <i>Ans.</i>	ft. in.
5184	2 6
	1 4
1536	
12	2 6
	10
1728) 18432 (10	
17280	3 4
	1 2
1152	
12	3 4
	6 8
1728) 13824 (8	
13824	3 10 8 <i>Ans.</i>

NOTE—This product, or solidity, is usually read 3 cubic feet 10 inches and 8 parts; but the 10 inches are $\frac{1}{12}$ of a unit of the 3, or $\frac{1}{2}$ of a cubic foot = 1440 cubic inches, and the 8 denotes $\frac{8}{12}$ or $\frac{2}{3}$ of a unit of the 10, and so on.

2. Find the solidity of a beam of timber, its length 12 feet 9 inches, breadth 1 foot 2 inches, and depth 1 foot 6 inches.

Ans. 22 feet 3 inches 9 p

3. A log of mahogany measures 8 feet 6 inches long, 3 feet 6 inches broad, and 4 feet 3 inches thick. Required its value at 12s. 9d. per cubic foot.

Ans. £80 12s. 0 $\frac{3}{4}$ d.

4. Find the content of 24 joists, each 15 feet 6 inches long, breadth 3 $\frac{1}{2}$ inches, and depth 9 $\frac{1}{2}$ inches.

Ans. 85 feet 10 inches 9 p.

5. Find the value of a marble slab, at 10s. 6d. per cubic foot, its length being 12 feet 6 inches, breadth 4 feet 6 inches, and thickness 1 foot 4 inches.

Ans. £39 7s. 6d.

6. Find the expense of digging a cellar, at 9d. per cubic yard, its length being 27 feet 6 inches, breadth 20 feet 6 inches, and depth 6 feet 6 inches.

Ans. £5 1s. 9 $\frac{1}{4}$ d.

7. How many gallons will a cistern hold, its length being 4 feet 2 inches, breadth 3 feet 6 inches, and depth 21 inches?

Ans. 159.0484 gallons.

CASE II.—To find the content of a round vessel or cylinder.

NOTE.—This rule will enable the plumber or mechanic to find the number of gallons that a pump will lift per stroke.

RULE.—Multiply the square of the diameter by .7854, and the product will be the area of the circle or bottom in square inches. Multiply this area by the depth of the vessel in inches, and the product will be the capacity in cubic inches. Divide this product by 277.274, and the quotient will be its content in gallons.

NOTE.—The reverse of this rule enables us to find the diameter of the base; having the content and depth given. See Example 2.

EXAMPLES.

A pump $4\frac{3}{4}$ inches in diameter, and 3 feet stroke. How many gallons does it pump each stroke?

Here $4\frac{3}{4} = 4.75$. Then $4.75 \times 4.75 \times .7854 = 17.7205875$ square inches area of the bottom.

And $17.7205875 \times 36 = 637.94115$ cubic inches, the capacity of the cylinder.

Therefore $637.94115 \div 277.274 = 2.30076$ gallons per stroke.

2. The content of a cylinder is 2.30076 gallons, and its depth 36 inches. Required the diameter of its base.

Here $2.30076 \times 277.274 = 637.94115$ cubic inches; and $637.94115 \div 36 = 17.7205875$ area of the base in square inches.

Therefore $\sqrt{17.7205875 \div .7854} = \sqrt{22.5625} = 4.75$ the diameter.

3. Find the expense of sinking a well, at 6s. 8d. per cubic yard; its depth being 15 yards, and the diameter 3 feet 6 inches.

Ans. £5 6s. 10½d.

4. A pump which has 4 feet stroke, and diameter $9\frac{1}{4}$ inches, makes 10 strokes per minute; how many gallons will it pump in 12 hours?

Ans. 88349.2147 gallons.

ALLIGATION.

ALLIGATION enables us to mix or compound several quantities of the same kind, but of different qualities, so that the compound or mixture may be of some intermediate quality or rate. It is usually divided into Alligation Medial and Alligation Alternate.

Alligation Medial being purely arithmetical; while Alligation Alternate, with its subdivisions, is a species of *Algebra*, called *Indeterminate Analysis* or *Unlimited Problems*, because they admit of an infinite number of different answers, for the finding of which Algebra furnishes us with a universal rule; the first portion of the subject is only given.

ALLIGATION MEDIAL.

RULE.—Multiply each of the given quantities by its rate, price, or quality, and collect the products into one sum, which divide by the sum of the quantities, and the quotient will be the value of a given quantity of the compound, in the same denomination of the multiples.

EXAMPLES.

1. Mixed 4 stones of flour, at 2s. 2d. per stone, with 5 stones at 2s. 4d. per stone, and 7 stones at 2s. 8d. per stone. Required the value of 1 stone of the mixture.

$s. \quad d.$	$s. \quad d.$	$d.$
$4 \times 2 \quad 2 = 8 \quad 8$	Or thus :	$4 \times 26 = 104$
$5 \times 2 \quad 4 = 11 \quad 8$		$5 \times 28 = 140$
$7 \times 2 \quad 8 = 18 \quad 8$		$7 \times 32 = 224$
16		16) 468
		12) 29½d.
		2s. 5½d.

2. Mixed 7 cwt. at 56s. per cwt., 9 cwt. at 48s., and 12 cwt. at 36s. Required the price per cwt. of the mixture.

Ans. 44s. 10½d.

3. Mixed 10 gallons of wine worth 14s. 6d. per gallon, with 20 gallons worth 15s. 6d. per gallon, and 30 gallons at 12s. 6d. per gallon. Required the selling price per gallon of the mixture, so as to gain 12½ per cent.

Ans. 15s. 6½d.

EXCHANGE.

EXCHANGE is the bartering of the money of one country for that of another. Its operations consist in ascertaining, by the Rule of Three, what sum of money of one country is equal to a given sum of any other country.

The real money of any country is certain pieces of metal, stamped by public authority to make them current; as sovereigns, half sovereigns, crowns, half crowns, shillings, &c., in England; francs, centimes, &c. in France.

THE COURSE OF EXCHANGE is such a variable sum of the money of one country as is given for a fixed sum of another; and the *Par of Exchange* is the sum of money which is equal to the fixed sum.

AGIO is the difference between Bank and Current money; the former being finer or purer than the latter. It is also applied to the difference between the actual value of the coin and that fixed by the government of the place that issued it.

USANCE is the time at which bills are drawn, varying according to contract, or custom of the place; that is, it is the time allowed before the bill is payable in cash.

FRANCE.

Paris gives to London from 25 to 26 francs for £1 sterling.
10 centimes make 1 decime, 10 decimes make 1 franc =
 $1\frac{1}{10}$ livre.

1. Exchange £250 sterling into francs, at 25 francs 50 centimes per £.

As £1 : £250 :: 25 fr 50 cen. : 6375 fr. *Ans.*

2. Exchange 6375 francs 40 centimes into English money, at 25 francs 40 centimes per £ sterling. *Ans.* £251.

AMSTERDAM.

Amsterdam gives to London a variable number of florins, from 11 to 13, as the case may be, for £1. 5 cents make 1 stiver, and 20 stivers make 1 florin.

1. Exchange £250 into florins, at 12 florins $2\frac{1}{2}$ stivers per £ sterling.

As £1 : £250 :: 12 flor. $2\frac{1}{2}$ sti. : 3031 $\frac{1}{2}$ flor. *Ans.*

2. Exchange 6062 $\frac{1}{2}$ florins into £ sterling, at 12 florins $2\frac{1}{2}$ stivers per £. *Ans.* £500.

Exchange is usually divided into Direct, Cross, and Indirect or Arbitrated.

Direct Exchange is the operation of exchange between two countries, without the medium of any other place.

Arbitration of Exchanges enables the merchant to compute the par of arbitration, by comparing it with the present course of Exchange in different countries, so as to know how to remit his money with the greatest advantage.

From the nature, extent, and importance of this subject. it is impossible, in the narrow limits of an Arithmetic, to do it justice. The Author begs to refer the student to Tate's Modern Cambist, a work of high respectability, written expressly on the subject.

R A T I O.

RATIO is the relation which one quantity bears to another of the same kind, numerically considered; the comparison being made by observing what multiple, part or parts, the one is of the other; thus 2 to 1, 4 to 2, and 12 to 6; generally written $2 : 1$, $4 : 2$, and $12 : 6$, and read two to one, four to two, &c.

The first term of the ratio is called the antecedent, and the second the consequent; and the magnitude of the ratio is found by dividing the antecedent by the consequent,—thus $\frac{4}{2} = 2$ the magnitude of the first: $\frac{12}{6} = 2$ the magnitude of the second; and $\frac{12}{6} = 2$ the magnitude of the third. Whence it appears that if the terms of any ratio be multiplied or divided by the same quantity, their ratio will not be changed.

The ratio is a ratio of greater or less inequality, when the antecedent is greater or less than its consequent; and it is a ratio of equality when the terms are equal: thus $9 : 6$, $6 : 9$, and $4 : 4$, or $5 : 5$. Then $\frac{9}{6} = \frac{3}{2}$ the magnitude of the first; $\frac{6}{9} = \frac{2}{3}$ the magnitude of the second; and so on.

CASE I.—*To compare the magnitudes of two or more ratios.*

RULE.—Find the magnitude of the ratios as above, and reduce them if required to a common denominator,

EXAMPLES.

1. Compare the magnitudes of the ratios of 4 : 5, and 6 : 7.

Here $\frac{4}{5}$ and $\frac{6}{7}$ are the magnitudes ;

$$\begin{array}{l} \text{Then } 4 \times 7 = 28 \\ \quad 6 \times 5 = 30 \\ \quad 5 \times 7 = 35 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{new numerators ;} \\ \\ \text{common denominator.} \end{array}$$

Therefore $\frac{30}{35} - \frac{28}{35} = \frac{2}{35}$; excess of $\frac{6}{7}$ over $\frac{4}{5}$;

Whence it follows, that the ratio of 6 : 7 is greater than the ratio of 4 : 5, or 6 has to 7 a greater ratio than 4 has to 5.

3. Compare the magnitudes of 2 to 3, and 5 : 6.

Ans. the 2nd is $\frac{1}{6}$ greater than the 1st.

CASE II.—*A ratio of greater inequality is diminished, and that of a less inequality increased, by adding the same quantities to both terms.*

Take 6 : 4 a ratio of greater inequality, and add 1 to each term ; then we shall have a new ratio of 7 : 5, by comparing these two ratios. 6 : 4 is equal to $\frac{6}{4}$ or $\frac{3}{2}$, and 7 : 5 is equal to $\frac{7}{5}$.

$$3 \times 5 = 15 \text{ new numerator for } \frac{3}{2};$$

$$7 \times 2 = 14 \quad \text{ditto} \quad \text{for } \frac{7}{5};$$

$$2 \times 5 = 10 \text{ common denominator ;}$$

$\therefore \frac{15}{10} - \frac{14}{10} = \frac{1}{10}$ the difference of the magnitudes of the two ratios ; whence 6 : 4 has a greater magnitude by $\frac{1}{10}$ than 7 : 5.

Again : let 5 : 7 be the ratio of less inequality, and add 2 to each term, then we shall have a new ratio of 7 to 9.

$$\therefore \frac{5}{7} = \frac{45}{63} \text{ the magnitude of the first ratio ;}$$

$$\text{and } \frac{7}{9} = \frac{49}{63} \quad \text{ditto} \quad \text{second ratio.}$$

Whence $\frac{49}{63} - \frac{45}{63} = \frac{4}{63}$ the quantity the ratio is increased by adding 2 to each of the terms.

CASE III.—*A ratio of greater inequality is increased, and that of a less inequality is decreased, by subtracting a like quantity from each of the terms.*

Take 6 : 4 a ratio of greater inequality, and subtract 1 from each term ; then the new ratio will be 5 : 3.

$$\text{Now the magnitude of the given ratio is } \frac{6}{4} = \frac{3}{2}$$

$$\text{and the magnitude of the new ratio is } \frac{5}{3} = \frac{5}{3}$$

Then $\frac{5}{3} - \frac{3}{2} = \frac{10}{6} - \frac{9}{6} = \frac{1}{6}$ or $\frac{1}{6}$, the quantity by which the magnitude of the new ratio exceeds the given one

Again, let $5 : 7$ be a ratio of less inequality; then by subtracting 2 from each term, we obtain a new ratio of $3 : 5$.

$\therefore$ the magnitude of the given ratio is $\frac{5}{7} = \frac{2\frac{1}{2}}{3\frac{1}{2}}$;
and that of the new one $\frac{3}{5} = \frac{2\frac{1}{5}}{3\frac{1}{5}}$;

Whence $\frac{2\frac{1}{2}}{3\frac{1}{2}} - \frac{2\frac{1}{5}}{3\frac{1}{5}} = \frac{4}{35}$ the quantity by which the magnitude of $5 : 7$ exceeds the magnitude of $3 : 5$.

CASE IV.—*The product of all the antecedents of any number of ratios, and the product of all the consequents, will form a new ratio.*

Thus, if there be taken $3 : 4$, $5 : 6$, and $7 : 8$; then will $3 \times 5 \times 7 : 4 \times 6 \times 8$; or $105 : 192$; and this ratio, compounded of the others, is called their sum.

EXAMPLES.

1. How much is the magnitude of the ratio of $7 : 5$ diminished by adding 2 to each of its terms? *Ans.* $\frac{4}{35}$

2. How much is the magnitude of the ratio of $5 : 7$ increased by adding 2 to each of its terms? *Ans.* $\frac{4}{63}$

3. How much is the magnitude of the ratio of $7 : 5$ increased by subtracting 2 from each term? *Ans.* $\frac{4}{15}$

4. How much is the magnitude of the ratio of $5 : 7$ diminished by subtracting 2 from each quantity? *Ans.* $\frac{4}{35}$

5. Find whether the ratios $6 : 7$, $9 : 11$, and $8 : 5$ are increased or decreased by adding 2, 3, and 4 to their terms respectively. *Ans.* 1st increased by $\frac{2}{63}$, 2nd, by $\frac{3}{77}$, and 3rd diminished by $\frac{4}{15}$

6. Find whether the ratios $7 : 5$, $8 : 11$, and $9 : 15$ are increased or diminished by subtracting 2, 4, and 6 from their terms respectively. *Ans.* 1st increased by $\frac{4}{15}$, 2nd decr. $\frac{1}{77}$, and 3rd decreased $\frac{4}{15}$.

7. What is the ratio arising from the composition of $4 : 5$ and $6 : 7$? *Ans.* $24 : 35$.

8. What is the ratio arising from the composition of $\frac{1}{2} : \frac{1}{3}$, $\frac{2}{3} : \frac{3}{4}$, $\frac{4}{5} : \frac{5}{6}$ and $4 : 5$? *Ans.* $\frac{16}{15} : \frac{5}{4}$.

PROPORTION.

PROPORTION is the relation of equality between two ratios : Thus, if we take the two ratios of equality, $4 : 6$ and $8 : 12$, the magnitude of the first is $\frac{2}{3}$, and that of the second $\frac{2}{3}$, and they are equal ; that is $4 : 6 = 8 : 12$, Therefore $\frac{4}{6} = \frac{8}{12}$. Whence $4 \times 12 = 6 \times 8$.

In Proportion, the product of the means is equal to the product of the extremes ; therefore $4 : 6 :: 8 : 12$.

In this proportion the 4 and 12 are the extremes, and the 6 and 8 the means.

In every proportion, the first term is greater, equal to, or less than the second, according as the third term is greater, equal to, or less than the fourth.

Two ratios not having the same magnitude, the four terms are not proportionals.

If four numbers constitute a proportion, the product of the means is equal to the product of the extremes ; hence, if the four terms of the proportion admit of a common divisor, each term may be divided by it, without destroying the proportion.

Thus in the proportion $16 : 24 :: 32 :: 48$;

Dividing by 8, then $2 : 3 :: 4 : 6$

For $\frac{16}{8} = \frac{2}{1}$, and $\frac{32}{8} = \frac{4}{1}$

NOTE.—For a more particular account of Proportion, see the Author's Algebra, or Dr. Wood's, by Lund.

GEOMETRICAL MEAN PROPORTION.

A geometrical mean proportion between any two numbers is the square root of their product. If it be required to find a geometric mean between 4 and 16 :

Here $4 \times 16 = 64$; then $\sqrt{64} = 8$, the geometric mean required. $\therefore 4 : 8 :: 8 : 16$.

To find two geometrical mean proportionals between two given numbers.

Divide the greater of the two given terms by the less, and the cube root of the quotient will be the common ratio of the terms. Multiply the first term by this ratio for the first mean, and the product of this mean and the ratio will be the second mean.

Find two mean proportionals between 4 and 32.

Here $32 \div 4 = 8$; then $\sqrt[3]{8} = 2$ the ratio. $\therefore 4 \times 2 = 8$ the first mean, and $8 \times 2 = 16$ the second mean. Hence 4, 8, 16, 32.

To find any number of geometrical means.

Divide the greater term by the less, and take such a root of the quotient as is denoted by the number of terms plus 1 ; i. e. if 3 means be required, the fourth root of the quotient must be taken ; if 4 means be required, the fifth root of the quotient must be taken ; and so on for the ratio. Multiply the 1st term by the ratio for the 1st mean, this mean by the ratio for the 2nd mean. and so on.

Required 3 geometrical means between 2 and 32.

Here $32 \div 2 = 16$; then $\sqrt[4]{16} = 2$ the ratio.

$\therefore 2 \times 2 = 4$ the first mean ; $4 \times 2 = 8$ the second ;
and $8 \times 2 = 16$ the third.

Hence 2, 4, 8, 16, 32.

PERMUTATION AND COMBINATION.

INTRODUCTION.—PERMUTATION is a changing of the relative situation or position of any given number of things, so that no two parcels or quantities of them shall have the quantities of which they are composed placed in the same order. Thus, if we take the letters *a* and *b*, they will permute, or change their position two different ways, as *ab* and *ba*, or $1 \times 2 = 2$.

Again, if we take three letters *a*, *b* and *c*, they may be permuted or changed six different ways, as *abc*, *acb*, *bca*, *cab*, and *cba* ; or $1 \times 2 \times 3 = 6$ different ways.

If four letters be taken, as *a*, *b*, *c* and *d*, then will $1 \times 2 \times 3 \times 4 = 24$ different ways, and so on for any number of quantities. Or, how many changes may be rung on four bells ? Thus, $1 \times 2 \times 3 \times 4 = 24$, *Ans.*

Permutation is sometimes called Changes or Variation.

COMBINATION is the several ways that a less number of things can be taken out of a greater. Thus, if we take two letters, *a* and *b*, only one combination can be formed, namely, *ab* or *ba*. If we take the quantities or letters *a*, *b* and *c*, we shall have only three combinations, by taking two and two, namely, *ab*, *ac*, and *bc* ; but they will form six permutations, namely, *ab*, *ba*, *ac*, *ca*, *bc*, and *cb*. If we take the four letters, *a*, *b*, *c*, *d*, and combine them two and two ; *a* may be combined as follows, *ab*, *ac*, *ad*, *bc*, *bd*, and *cd*, thus we have six combinations of the four quantities, two together.

But, for their permutation, we must write each combination twice, viz. *ab* and *ba*, &c., which will make twelve permu-

tations, when two are taken together: as how many changes may be rung with 2 bells out of 4?

$$4 \times (4 - 1) = 4 \times 3 = 12.$$

PROBLEM I.—*To find the number of permutations, or changes [that can be made of any given number of different things.*

RULE*—Multiply all the terms of the given series continually together, and the last product will be the number of permutations or changes required.

1. How many changes may be rung on 6 bells?

Here $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ *Ans.*

2. How many days can 8 persons be placed in a different position at dinner? *Ans.* 40320 days.

3. There are 13 bells in the Parish Church, Leeds. How many changes can be rung on the same; and what time will it require, allowing 10 changes per minute, and the year to consist of 365 days, 5 hours and 49 minutes?

Ans. 6227020800 changes, and 1183 years, 350 days, 6 hours, 53 min. 11 sec.

PROBLEM II.—*Given any number of different things, to find how many permutations or changes can be made out of them, by taking a given number of them at a time.*

RULE.—Take a series of numbers, beginning at the given number of things, and decreasing by 1 to the number of quantities to be taken at a time, and the continued product of all the terms will be the answer required.

1. How many changes may be rung upon 2 bells out of 8; that is taking 2 at a time?

Here $8 \times (8 - 1) = 8 \times 7 = 56$ *Ans.*

2. How many changes may be rung with 3 bells out of 8?

Here 3 at a time are taken,

Then $8 \times (8 - 1) \times (7 - 1) = 8 \times 7 \times 6 = 336$ *Ans.*

PROBLEM III.—*To find how many combinations may be made out of a given number of different things, by taking a given number at a time, so that no two sets may be alike.*

RULE—Take the numbers 1, 2, 3, &c. up to the number of things to be taken at a time, and find their product.

* The reason of this rule is obvious from the introduction.

Take a series of as many terms, as the number of things to be taken at a time, beginning with the given number out of which the election is to be made, decreasing each succeeding term by 1, and find their product; divide this product by the former, and the quotient will be the number of combinations required.

1. How many combinations can be made of 4 letters out of 9? Here, 4 are to be taken at a time;

then, $1 \times 2 \times 3 \times 4 = 24$, for the divisor.

Again, 9 is the given number, and as 4 is to be taken out of them at a time, we must have a series of four numbers beginning with 9 as follows:

$$9 \times (9-1) \times (8-1) \times (7-1)$$

Or, $9 \times 8 \times 7 \times 6 = 3024$, the dividend.

$24 \mid 3024 \quad 126$ the number of combinations.

2. How many days may 10 men dine, taking 6 at a time, without the same company coming a second time?

Ans. 210 days.

PROBLEM IV.—*Given a number of things, in which there are several of one sort and several of another, to find how many permutations or changes can be made out of them.*

RULE.—Take the series 1, 2, 3, 4, &c. up to the given number of things, and find their product.

Take the series 1, 2, 3, 4, &c. up to the given number of things of the first sort, and the series 1, 2, 3, 4, &c. up to the given number of things of the second sort, and so on.

Divide the product of all the terms of the first series by the joint product of all the terms of the remaining series, and the quotient will be the number of permutations or changes required.

1. How many variations can be made of the letters in the word Bacchanalia?

Here are 11 letters in the word,

$$\text{Then } 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 = 39916800;$$

Again, we have 4 *a*'s and 2 *c*'s;

$$\text{Then } 1 \times 2 \times 3 \times 4 = 24 \text{ for the } a\text{'s,}$$

$$\text{and } 1 \times 2 = 2 \text{ for the } c\text{'s.}$$

$$\text{Then } 24 \times 2 = 48;$$

$$\text{Therefore } 39916800 \div 48 = 831600 \text{ } \textit{Ans.}$$

2. How many variations can be made of the letters in the word Teetotalism?

Ans. 3326400.

APPENDIX.

MISCELLANEOUS QUESTIONS AND OBSERVATIONS

Explanatory to the Principles of Arithmetic ; together with Demonstrations to some of the Principal Rules.

1. Reduce $\frac{15}{20}$ to an equivalent fraction in its lowest terms.

Here, by Case I, page 88, 5 will be the greatest common measure ; then $\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$. By this expression we understand that the unit or quantity

which it represents, is to be divided into 4 equal parts, and that the numerator 3 signifies that 3 of these parts are to be taken. If we take £1 for the unit, then $20s. \div 4 = 5s.$, which is $\frac{1}{4}$ of a £ ; then $5s. \times 3 = 15s. = \frac{3}{4}$ of a £.

Now, as this $\frac{3}{4}$ is derived from $\frac{1}{2} \times \frac{3}{2}$; as before, take $\frac{1}{2} \times \frac{3}{2}$ in the denomination of £, i. e. take £1 for the unit, which divide into 20 equal parts, one of which will be 1s. ; then $1s. \times 15 = 15s. = \frac{3}{4}$ of a £, as above ; hence, it appears that if the numerator and denominator of a fraction be divided by the same number, the fraction, though in less terms, still retains its value.

Again, if we multiply the numerator and denominator by the same number, as $\frac{15 \times 12}{20 \times 12} = \frac{180}{240}$. Now, if we consider this fraction in the denomination of a £ ; or, in other words, take £1 for the unit, and divide this £1 into 240 equal parts, each part will be 1d. ; then $1d. \times 180 = 180d. = 15s. = \frac{3}{4}$, as derived from the $\frac{1}{2} \times \frac{3}{2}$ or $\frac{3}{4}$. Consequently, if the numerator and denominator of the same fraction be either multiplied or divided by the same number, the value of the fraction is neither increased nor decreased.

2. Reduce $2\frac{1}{4}$ to an improper fraction. Here, as any number may be converted into a fraction by placing 1 for its denominator, $2\frac{1}{4}$ may be represented by $\frac{2\frac{1}{4}}{1}$, and as multiplying the numerator and denominator by the same number,

neither increases nor decreases its value, we have $\frac{2\frac{1}{4} \times 4}{1 \times 4} = \frac{9}{4}$. *Ans.*

3. If it be required to reduce $\frac{9}{4}$ to its equivalent mixed number. Here $\frac{9 \div 4}{4 \div 4} = \frac{2\frac{1}{4}}{1} = 2\frac{1}{4}$. If we take $2\frac{1}{4}$ in the denomination of £1, its value is evidently £2 5s. Now, as the $\frac{9}{4}$ are derived from $2\frac{1}{4}$, let $\frac{9}{4}$ be in the denomination of a £, or take £1 for the unit ; then $\frac{1}{4}$ of a £ is 5s., and $5s. \times 9 = 45s. = £2\ 5s.$

4. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction. Here, if we divide the $\frac{4}{5}$ into 3 equal parts, as denoted by the denominator 3, in the first fraction, then one of these equal parts is $\frac{1}{5}$, then $\frac{1}{5} \times 2 = \frac{2}{5}$. Again, divide $\frac{2}{5}$ into 4 equal parts,

as denoted by the denominator 4 in the $\frac{2}{4}$, the result will be $\frac{1}{2}$; then $\frac{1}{2} \times 2$ (the numerator in the last found fraction), the result is $\frac{2}{2}$, the single fraction required. [The same result may be found by Case 4th, page 90.]

5. Reduce $\frac{2}{3}$, $\frac{1}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to fractions of the same value, having a common denominator. [See solution 2nd to Example 1st, page 93.]

And, we observe that the numerators and denominators of the different fractions are multiplied by the same numbers; consequently, their values are not changed. Fractions may, in some cases, be more readily reduced to a common denominator, by multiplying the terms of some and dividing those of others by some convenient number: thus, if we take $\frac{2}{3}$, $\frac{5}{9}$, $\frac{1}{18}$, and $\frac{9}{27}$. Here $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$, $\frac{10 \div 2}{18 \div 2} = \frac{5}{9}$ and $\frac{9 \div 3}{27 \div 3} = \frac{3}{9}$, hence the given fractions become $\frac{6}{9}$, $\frac{5}{9}$, $\frac{1}{9}$, and $\frac{3}{9}$, with a common denominator. When fractions are reduced to a common denominator, their values are readily compared.

6. If we have to compare $\frac{2}{3}$ and $\frac{3}{4}$. Here $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$, by this we see that the first fraction is less than the second; in the first, the unit or quantity which it represents is to be divided into 12 equal parts, 8 of them are taken, but in the second, 9 of these equal parts are required; hence it is evident, of two fractions having the same denominator, that is the greater which has the greater numerator; also, of any two fractions having the same numerator, that is the less which has the greater denominator.

7. Reduce $\frac{4}{5}$ to a fraction of the same value, having a given numerator or denominator. In the first place, let it be reduced to an equivalent fraction, having 12 for its numerator. Here $\frac{4}{5}$ denote the ratio of 4 : 5, hence it follows that we have to find a number that 12 shall bear the same ratio to that 4 does to 5, for a denominator to the new fraction. $\therefore 4 : 5 :: 12 : 15$; hence $\frac{12}{15}$ is the fraction required. If it be required to reduce $\frac{4}{5}$ to an equivalent fraction, having 20 for its denominator; here we have to find a number that has the same ratio to 20 for a numerator that 5 has to 4; hence $5 : 4 :: 20 : 16$; $\therefore \frac{16}{20}$, the new fraction required.

8. Reduce $\frac{3}{5}$ of a £ to the fraction of a guinea. Here the given quantity, $\frac{3}{5}$, is in the denomination of a £, and may be written £ $\frac{3}{5}$. Now, to bring pounds into guineas, they must first be brought into shillings, and then divided into guineas; thus $\frac{3 \times 20}{5}$ denotes shillings, and if we divide this quantity by 21, i.e. multiply the denominator by 21, the result will be guineas; hence $\frac{3 \times 20}{5 \times 21} = \frac{60}{105} = \frac{4}{7}$ guineas. Now, if we find the value of $\frac{3}{5}$ of a £, and also $\frac{4}{7}$ of a guinea; here the unit of the first fraction will be £1, and that of the second 1 guinea.

5) 20s.

4
3

12s. = $\frac{3}{5}$ of a £.

7) 21s.

3
4

12s. = $\frac{4}{7}$ of 1 guinea.

Hence it appears that $\frac{3}{5}$ of a £ is of the same value of $\frac{4}{7}$ of a guinea.

9. In order to find the sum or difference of two fractions, they must be reduced to a common denominator.

Required the sum and difference of $\frac{2}{3}$ and $\frac{1}{6}$. Here $\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$, because multiplying the numerator and denominator of the fraction does not alter its value: $\therefore$ to find their sum, $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$, and to find their difference $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$.

To find the sum and difference of 1 and $\frac{7}{8}$. Here $1 = \frac{8}{8}$, then $\frac{1 \times 8}{1 \times 8} = \frac{8}{8}$; $\therefore \frac{8}{8} + \frac{7}{8} = \frac{15}{8}$, their sum, and $\frac{8}{8} - \frac{7}{8} = \frac{1}{8}$ the difference.

10. To multiply a vulgar fraction by a given number, is to take the numerator as many times as there are units in the given number or multiplier, and place the given denominator under the product; or divide the denominator of the fraction by the multiplier; thus multiply $\frac{4}{15}$ by 5. Here $\frac{4 \times 5}{15 \times 1} = \frac{20}{15} = \frac{4}{3}$; or, $\frac{4}{15 \div 5} = \frac{4}{3}$.

11. To divide a vulgar fraction by a given number; it is the same, if we divide the numerator or multiply the denominator by the given divisor.

Divide $\frac{6}{7}$ by 3. Here $\frac{6 \div 3}{7} = \frac{2}{7}$ or $\frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}$

Hence it follows, that to multiply a fraction, we multiply the numerator; and to divide a fraction we multiply the denominator.

12. Simplify the following expressions:—

$$1. \quad \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6}. \quad \text{Ans. } \frac{23}{60}$$

$$2. \quad \frac{4}{5} + \frac{5}{7 \text{ of } \frac{7}{8}} \quad \text{Ans. } 7 \frac{7}{5}$$

$$3. \quad \frac{2}{3} \times [12 - \frac{2}{3} \text{ of } \frac{3}{5}] \div \frac{3}{4} \times [8 + \frac{3}{5} \text{ of } 6\frac{1}{2}] \quad \text{Ans. } \frac{228}{1071}$$

13. In Decimal Fractions, commonly called Decimals, the numerator only is written; the denominator being 1 with as many ciphers annexed to the right of it as there are decimal places; thus $.5 = \frac{5}{10}$, $.25 = \frac{25}{100}$, and $.005 = \frac{5}{1000} = \frac{5}{1000}$ Then

$$\begin{array}{r} .5 \\ .25 \\ .005 \\ \hline .755 \text{ Sum.} \end{array}$$

In order to find the sum of the above Decimals, and retain their denominators, we must, as in Vulgar Fractions, reduce them to a common denominator; thus $\frac{5}{10} = \frac{500}{1000}$ and $\frac{25}{100} = \frac{250}{1000}$; therefore $\frac{500}{1000} + \frac{250}{1000} + \frac{5}{1000} = \frac{755}{1000} = .755$; which shews that, in the addition of decimals, the figures must be so placed that those of the same denomination must stand under each other. The same remark will hold good for Subtraction of Decimals.

14. Multiply .56 by .24, also by .004, and shew that the product must contain as many decimal places as both the multiplier and multiplicand ; and if there be not so many places in the product, the defect must be made up by annexing ciphers to the left of it.

$$\begin{array}{r}
 .56 \\
 .24 \\
 \hline
 224 \\
 112 \\
 \hline
 .1344
 \end{array}
 \qquad
 \begin{array}{r}
 .56 \\
 .004 \\
 \hline
 .00224
 \end{array}$$

By placing the denominators to these decimals, *i. e.* reducing them (Case 2) to their equivalent fractions, thus $.56 = \frac{56}{100}$, and $.24 = \frac{24}{100}$ then $\frac{56}{100} \times \frac{24}{100} = \frac{1344}{10000} = .1344$. Also $.56 = \frac{56}{100}$, and $.004 = \frac{4}{1000}$ or $\frac{4}{1000}$; then $\frac{56}{100} \times \frac{4}{1000} = \frac{224}{100000} = .00224$. Now, as the denominator contains 1 and five ciphers annexed to the right hand of it, this indicates that we must have five places of decimals ; and that the first two stands in the thousands, the second two in the tens of thousands and the four in the hundreds of thousands place. $\therefore \frac{224}{100000} = .00224$.

Multiplying a decimal by 10 is only removing the decimal point one place to the right ; multiplying by 100 is only removing the point two places to the right, and so on.

15. Reduce $\frac{4}{9}$ of a farthing to the decimal of $\frac{4}{9}$ of a guinea. *Ans.* .02083

16. What decimal is equivalent to the sum of .56, .015, and $\frac{1}{1000}$?
Ans. .725

17. Find the sum of $\frac{2}{3}$ of $\frac{3}{5}$ of £3 12s., $\frac{2}{3}$ of $\frac{4}{5}$ of £1 14s., and $\frac{2}{3}$ of 18 half crowns, and reduce the result to the decimal of £10. *Ans.* .224

18. Express the difference between .78 cwt and .987 oz.
Ans. 3 qr. 3 lb. 4 oz. 12,368 dr.

19. Find the sum, difference, product, and quotient of .428571 and .8
Ans. 1.317460 sum ; .460317 difference ; 380952 product ; and .482142857 quotient.

20. Find what decimal part of a guinea is 3s 7 $\frac{1}{2}$ d and $\frac{2}{3}$ of a farthing.
Ans. .1742063492

21. Find a vulgar fraction equivalent to the continued product of 0.345, .18, and .59.
Ans. $\frac{114}{3025}$

22. Find the cube root of $2 + \frac{1}{2} + \frac{6}{300} + \frac{8}{20000}$. *Ans.* 1.412874

Multiplication being only a short method of Addition ; so Division is only an abbreviation of Subtraction.

If it be required to multiply 365 by 73, here we should have to take 365 73 times, and add up the sum as in Simple Addition.

Divide 365 by 73.
 $73 \overline{) 365}$ (5 quotient.
 365
 —

Or thus, $\frac{365}{73} = 5$
 $\frac{292}{73} = 4$
 $\frac{219}{73} = 3$
 $\frac{146}{73} = 2$
 $\frac{73}{73} = 1$

DEMONSTRATIONS, &c.

Prove the rule for casting out the nines in multiplication.

Let $9x + y$ and $9a + b$ denote the multiplier and multiplicand ; then $81ax + 9bx + 9ay + by$ will be their product ; here the three leading numbers contain an exact number of nines, it is evident that the excess of 9's in the fourth term by will be the same as the excess of 9's in the whole product ; but b and y are the excess of 9's in the multiplier and multiplicand, and by is their product ; hence the proof is obvious.

TO THE RULE OF THREE.

Take $a : b = c : d$; or $a : b :: c : d$; prove that $ad = bc$.

Let $r =$ the common ratio ; then $\frac{a}{b} = r$, and $\frac{c}{d} = r$; multiplying these equations by bd , $ad = bdr$, and $bc = bdr$, $\therefore ad = bc$.

TO THE RULES OF INTEREST, PRESENT WORTH & DISCOUNT.

To find the simple interest and amount of a given sum, for a given time, at a given rate.

Let $P =$ the principal, or sum lent ; r the rate of interest of £1 for 1 year ; n the number of years ; i the interest of the sum lent, viz. of P £. ; M the amount of P ; then $rP =$ the interest of P £ for 1 year ; and $nrP =$ the interest of P £. for n years ; wherefore we shall have $i = nrP$ and $M = P + nrP = (1 + nr)P$. We must bear in mind that r is only the interest of £1 for 1 year ; if however R denote the rate per cent ; then $\frac{R}{100} =$ rate of interest of £1 for 1 year, $\therefore r = \frac{R}{100}$; hence, by substitution, $i = \frac{nRP}{100} \dots$

(A) and $M = \left(1 + \frac{nR}{100}\right) P \dots (B)$.

The above two formulæ will enable us to solve every question relating to Simple Interest. The expression denoted by (A) may, however, be further altered into $i = \frac{P \times R}{100} \times n$, which gives the rule. Multiply the principal by the rate per cent. ; divide the product by 100, and the quotient will be the interest for 1 year, which being multiplied by n gives the interest for n years." The equation denoted by (B) may also be further altered into

$M = P + \left(\frac{P \times R}{100}\right)n$, which denotes that the interest added to the principal gives the amount. The student must bear in mind, in the equation

$M = \left(1 + \frac{nR}{100}\right)P = \left(\frac{100 + nR}{100}\right)P$, that M is the amount of a sum of money P , put out to interest for n years, at R per cent. Hence, to find such a sum P which, being put to interest at a given rate, shall, after a certain time, amount to a given sum M , we shall have the following formula:—

$\frac{P}{M} = \frac{100}{100 + nR}$ and $\therefore$ we obtain the following proportion: As 100 $+ nR : 100 :: M : P$, and this put into words, gives the following rule for the present worth. "As the amount of £100 for the given rate and time is to £100, so is the given amount to the present worth required.

A little further consideration will enable us to determine the Rule for Discount:—For as $\frac{P}{M} = \frac{100}{100 + nR}$ $\therefore$ also $\frac{100 + nR}{100} = \frac{M}{P}$; $\therefore$ also $\frac{100 + nR}{100} - 1 = \frac{M}{P} - 1$; $\therefore$ also $\frac{nR}{100} = \frac{M - P}{P}$; $\therefore$ also $\frac{100}{nR} = \frac{P}{M - P}$; $\therefore$ also $\frac{100}{nR} + 1 = \frac{P}{M - P} + 1$; $\therefore$ also $\frac{100 + nR}{nR} = \frac{M}{M - P} = \frac{M}{D}$... (C) as the difference between the amount and present worth gives the discount $= D$ suppose.

The equation (C) gives the following proportion:—As $100 + nR : nR :: M : D$, which being put into words, gives therefore the following Rule:—As the amount of £100, for the given time, at the given rate, is to the interest of £100 for that time, so is the proposed sum to the discount required.

TO THE RULE FOR SQUARE ROOT.

Let mn be any number expressed as in common arithmetic; then n denotes the digit in the units place, and m the digit or digits in the place of the tens, such that the digits of which n is made may equal any number whatever. Then $mn = 10m + n$; $\therefore (mn)^2 = (10m + n)^2 = (m^2 \times 100 + 2m \times 10 \times n + n^2) = m^2 \times 100 + (2m \times 10 + n)n$. Then if we denote $(mn)^2$ by N ; $N = m^2 \times 100 + (2m \times 10 + n)n$, and $\sqrt{N} = 10m + n$ or mn by arithmetical notation; it is evident that $\sqrt{N}$ is obtained by the following process: Find the greatest square in N and subtract it from N ; then divide the remainder by twice the root found, (which is that number which being multiplied by itself gives the greatest square in N), together with the remainder of the root added; and after division there will be no remainder if N be a complete square. The work being exhibited will run thus—

$$\begin{array}{r}
 N(m \times 10 + n) \\
 m^2 \times 100 \\
 \hline
 2m \times 10 + n) \quad N^1 \quad = \quad 2m \times 10 + n) n \\
 \quad \quad \quad N^1 \\
 \hline
 \end{array}$$

$$\text{Or, } \frac{N}{m^3 \times 100} \quad (m n)$$

$$2 m n) \frac{N^1}{N^1}$$

As $m \times 10 + n = m n$, and $2 m \times 10 + n = 2 (m n)$ in arithmetic.

TO THE RULE FOR CUBE ROOT.

Let $m n$ represent any number as before. Then $(m n)^3 = (10 m + n)^3 = m^3 \times 1000 + [3 (10 m)^2 n + 3 (10 m) n^2 + n^3] \therefore N = (m^3 \times 1000) + [m^2 \times 300 \times n \times (n^2 + m \times 30 + n^3)]$ if N represent $m n$.

Now it is easy to see that the ordinary rule for Cube Root is derived from the above. For exhibiting the work,

$$\begin{array}{r} N (m \times 10 + n \text{ or } m n) \\ m^3 \times 1000 \\ \hline - m^2 \times 300) \quad N^1 \quad \text{this gives } n \\ \hline m^2 \times 300 \times n \\ n^2 \times m \times 30 \\ \quad n^3 \\ \hline N^1 \end{array}$$

For $N - (m^3 \times 1000) = [m^2 \times 300 \times n + (n^2 \times m \times 30) + n^3]$ $= N^1$ suppose.

Again, let $\sqrt{N} = a$, and $\therefore N = a^2$. Let then a become $a + 1$, then N becomes $N^1 = a^2 + 2a + 1$; i.e. the root cannot be increased by 1, but at the same time its square is increased by twice the original root + 1. If therefore at any time the remainder is not greater than twice the root already found, the root so obtained may be correct as far as it has gone.

In extracting the square root of a number which has $2n + 1$ figures in its root, then when $n + 1$ figures have been found, the remaining n figures may be found by division. For if $a + b$ be the root of $2n + 1$ figures whereof a contains $n + 1$ and b , n figures, and $\therefore a$ followed by n ciphers; then $(a + b)^2 = a^2 + 2ab + b^2$; $\therefore$ after the greatest square has been taken from $(a + b)^2$, the remainder is $2ab + b^2$, which being divided by $2a$ gives $b + \frac{b^2}{2a}$ for the quotient, differing from b by only $\frac{b^2}{2a}$; but as b consists of n figures, b^2 cannot comprise more than $2n$ figures, and a comprises $n + 1$ figures followed by n ciphers, or $2n + 1$ figures on the whole; $\therefore \frac{b^2}{2a}$ is a proper

fraction ; hence, after a , consisting of $n + 1$ figures, has been found, the remaining n figures may be obtained simply by dividing by $2a$. (See question page 175.)

When the cube consists of $2n + 2$ figures, and $n + 2$ figures of the root have been found, the remaining n figures may be found by division ; i.e. divide the remainder with ciphers annexed by three times the square of the figures in the root already found, for the remaining part of the root.

Let $a + b$ be the root, where a comprises $n + 2$ figures followed by ciphers, and b composes the remaining n figures ; then when of $\sqrt[3]{N}$, a is determined, the remainder is $3a^2b + 3ab^2 + b^3$, which being divided by $3a^2$ gives for the quotient $b + \frac{b^2}{a} + \frac{b^3}{3a^2}$, differing from b by only $\frac{b^2}{a} + \frac{b^3}{3a^2}$, but b^2 which, as b contains only n figures, must be less than $(10)^{2n}$ which contains $2n + 1$ figures, and a by supposition is not less than $(10)^{2n+1}$; $\therefore \frac{b^2}{a}$ cannot be greater than $\frac{1}{10}$; also b^3 must be less than $(10^n)^3$, or $(10)^{3n}$, and $3a^2$ is not less than $3 \times 10^{4n+2}$; therefore $\frac{b^3}{3a^2}$ cannot be greater than $\frac{1}{10^{n+2}}$; whence it follows that the error cannot be greater than $\frac{1}{10} + \frac{1}{10^{n+2}}$; i.e. than $\frac{1}{10} + \frac{1}{10^2}$ or $\frac{11}{100}$ whatever n may be ; consequently the last n figures will be accurately obtained by Division. (See Example page 179.)

TO RULES 2ND AND 3RD, PAGE 129.

Find the interest of £219 for 50 days, at 5 and $4\frac{1}{2}$ per cent. per annum.

Here $\frac{219 \times 5}{100}$ the interest for 1 year at 5 per cent.

Then, as 365 days : 50 days :: $\frac{219 \times 5}{100}$ £ : $\frac{219 \times 5 \times 50}{365 \times 100} = \frac{219 \times 50}{73 \times 100}$ or $\frac{219 \times 50}{7300}$ the interest at 5 per cent., for 50 days, when reduced = £1 10s., hence follows the 2nd Rule.

Again, if we multiply the interest at 5 per cent. by 2 and divide the product by 10 the result will be the interest at 1 per cent. Thus, $\frac{219 \times 50 \times 2}{7300 \times 10}$; $\therefore \frac{219 \times 50 \times 2 \times 4\frac{1}{2}}{73000} = \frac{98550}{73000} =$ £1 7s. interest, at $4\frac{1}{2}$ per cent. Hence follows Rule 3rd





